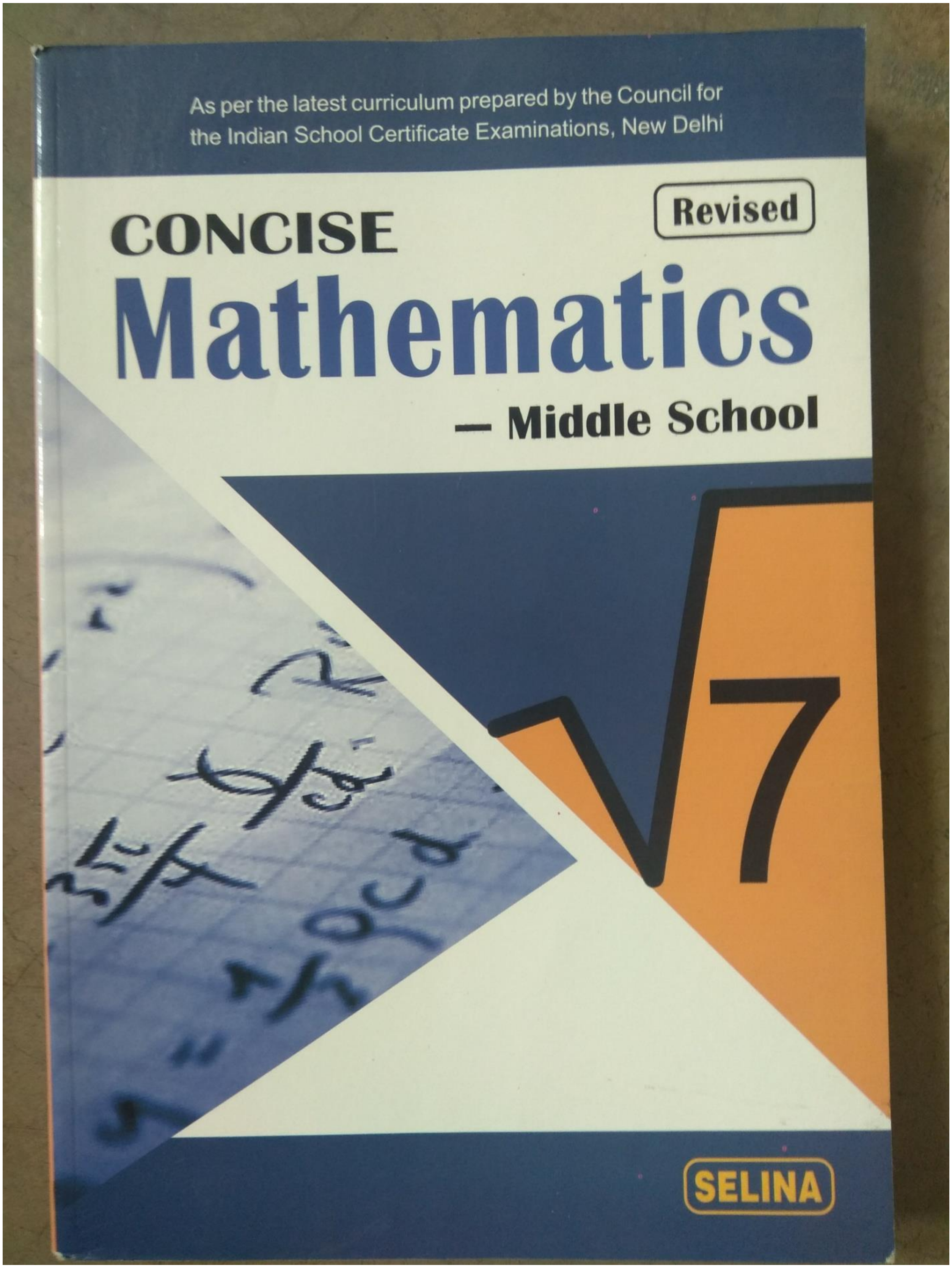


As per the latest curriculum prepared by the Council for
the Indian School Certificate Examinations, New Delhi

Revised

CONCISE
Mathematics
— Middle School

$\sqrt{7}$

SELINA

Contents

Theme – 1
NUMBERS SYSTEM

INTEGERS 1

1.1 INTRODUCTION

Integers (I) = ..., –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ...

= ...., –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ...

Negatives of natural numbers ↑ Zero Natural Numbers

Positive integers = I^+

= 1, 2, 3, 4, 5, ...

Negative integers = I^-

= ...., –8, –7, –6, –5, –4, –3, –2, –1

= Negatives of natural numbers

1. Positive integers go upto infinity on the right side of 0 (zero) *i.e.* on positive side.
2. Negative integers go upto infinity on the left side of 0 (zero) *i.e.* on negative side.
3. Zero (0) is simply an integer. It is neither negative nor positive.

1.2 MULTIPLICATION OF INTEGERS

1. *The product of two integers of the same sign is always positive.*

   i.e. (one positive integer) × (another positive integer) = a positive integer

   and (one negative integer) × (another negative integer) = a positive integer

(+ve) × (+ve) = + ve

and (–ve) × (–ve) = +ve.

For example :

(i) 5 × 4 = 20 and (–5) × (–4) = 20

(ii) 7 × 6 = 42 and –7 × – 6 = 42

(iii) 15 × 8 = 120 and (–15) × (–8) = 120 and so on.

2. *The product of one positive integer and one negative integer is always negative.*

   i.e. (one positive integer) × (one negative integer) = a negative integer

   and (one negative integer) × (one positive integer) = a negative integer

(+ve) × (–ve) = – ve

and (–ve) × (+ve) = – ve.

For example :

(i) 5 × (–4) = –20 and (–5) × 4 = –20

(ii) 7 × (–6) = –42 and (–7) × 6 = –42

(iii) 15 × (–8) = –120 and (–15) × 8 = –120 and so on.

1.3 PROPERTIES OF MULTIPLICATION OF INTEGERS

1. Closure Property :

The multiplication (product) of two integers is always an integer.

That is : If m and n are integers, then $m \times n$ *i.e.,* mn is also an integer.

For example :

(i) 5 is an integer and 6 is an integer, then their multiplication
i.e., $5 \times 6 = 30$ is also an integer.

(ii) 8 is an integer and –12 is an integer, then their multiplication $8 \times -12 = -96$ is also an integer.

(iii) –15 and –6 are integers, then $(-15) \times (-6) = 90$ is also an integer.

(iv) –13 and 8 are integers and so $(-13) \times 8 = -104$ is also an integer and so on.

2. Commutative Property *i.e.* Commutativity :

According to this property, if m and n are two integers, then $m \times n = n \times m$

Consider the following table :

	m	n	$m \times n$	$n \times m$	Is $m \times n = n \times m$?
1.	6	7	$6 \times 7 = 42$	$7 \times 6 = 42$	Yes
2.	–10	8	$(-10) \times 8 = -80$	$8 \times (-10) = -80$	Yes
3.	–15	–7	$(-15) \times (-7) = 105$	$(-7) \times (-15) = 105$	Yes
4.	21	–6	$21 \times (-6) = -126$	$(-6) \times 21 = -126$	Yes

3. Associative Property *i.e.* Associativity :

According to this property, if l, m and n are three integers, then $l \times (m \times n) = (l \times m) \times n$

Consider the following table :

	l	m	n	$l \times (m \times n)$	$(l \times m) \times n$	Is $l \times (m \times n) = (l \times m) \times n$?
1.	5	3	2	$5 \times (3 \times 2) = 5 \times 6 = 30$	$(5 \times 3) \times 2 = 15 \times 2 = 30$	Yes
2.	–5	3	2	$-5 \times (3 \times 2) = -5 \times 6 = -30$	$(-5 \times 3) \times 2 = -15 \times 2 = -30$	Yes
3.	8	–5	4	$8 \times (-5 \times 4) = 8 \times (-20) = -160$	$(8 \times -5) \times 4 = -40 \times 4 = -160$	Yes
4.	4	–3	–5	$4 \times (-3 \times -5) = 4 \times 15 = 60$	$(4 \times -3) \times -5 = -12 \times -5 = 60$	Yes

4. Distributive Property (Distributivity) :

According to this property, if l, m and n are three integers, then

$$l \times (m + n) = l \times m + l \times n$$

i.e. multiplication is distributive over addition.

Consider the following table :

	l	m	n	
1.	5	7	2	$l \times (m + n) = 5 \times (7 + 2) = 5 \times 9 = 45$ and $l \times m + l \times n = 5 \times 7 + 5 \times 2 = 35 + 10 = 45$ $\therefore\ l \times (m + n) = l \times m + l \times n$

2.	8	–9	5	$l \times (m + n) = 8 \times (-9 + 5) = 8 \times -4 = \mathbf{-32}$ and, $l \times m + l \times n = 8 \times -9 + 8 \times 5 = -72 + 40 = \mathbf{-32}$ $\therefore\ \boldsymbol{l \times (m + n) = l \times m + l \times n}$
3.	–6	–15	8	$l \times (m + n) = -6 \times (-15 + 8) = -6 \times -7 = \mathbf{42}$ $l \times m + l \times n = -6 \times -15 + (-6) \times 8 = 90 - 48 = \mathbf{42}$ $\therefore\ \boldsymbol{l \times (m + n) = l \times m + l \times n}$

Note 1 : Distributivity of multiplication over addition can also be stated as :
$(l + m) \times n = l \times n + m \times n.$

Note 2 : Distributivity of multiplication over subtraction is also true :
i.e. $l \times (m - n) = l \times m - l \times n$
and $(l - m) \times n = l \times n - m \times n.$

For example :

If $l = 8$, $m = 13$ and $n = 7$, then

$$l \times (m - n) = 8 \times (13 - 7)$$
$$= 8 \times 6 = \mathbf{48}$$
and $$l \times m - l \times n = 8 \times 13 - 8 \times 7$$
$$= 104 - 56 = \mathbf{48}$$
$$\therefore \quad \boldsymbol{l \times (m - n) = l \times m - l \times n}$$

5. Existence of multiplicative identity :

For every integer ***a***, we have

$\boldsymbol{a} \times 1 = \boldsymbol{a}$ and $1 \times \boldsymbol{a} = \boldsymbol{a}$

i.e. $\boldsymbol{a} \times 1 = 1 \times \boldsymbol{a} = \boldsymbol{a}$

∵ Multiplication of every integer ***a*** with integer one (1) gives the integer ***a*** itself.

Integer 1(one) is called the multiplicative identity.

For example :

$5 \times 1 = 5$, $1 \times 5 = 5$, $-12 \times 1 = -12$, $1 \times (-12) = -12$ and so on.

6. Existence of multiplicative inverse :

For any integer ***a***, its multiplicative inverse will be $\frac{1}{a}$ so that $a \times \frac{1}{a} = 1$, the multiplicative identity.

Thus, the multiplicative inverse of an integer exists if :
the integer × its multiplicative inverse = 1, the multiplicative identity.
Out of all the integers :

(i) multiplicative inverse of 1 is 1 itself as $1 \times \frac{1}{1} = 1$

(ii) multiplicative inverse of –1 is –1 itself as $-1 \times \frac{1}{-1} = 1$

7. (i) $-2 \times -3 \times -4$
$= -24$ [Product of *three* negative integers]

(ii) $-2 \times -3 \times -4 \times -5 \times -6$
$= -720$ [Product of *five* negative integers]

(iii) $-2 \times -3 \times -4 \times -5 \times -6 \times -7 \times -8$
$= -40320$ [Product of *seven* negative integers]

In (i), given above, we have multiplied three negative integers and we found that the product is a negative integer.

In (ii), given above, multiplication of 5-negative integers gives a negative integer.
In (iii), given above, multiplication of 7-negative integers gives a negative integer.
Thus, we conclude that *the proudct of odd number of negative integers is always negative.*

8. (i) $-2 \times -3 = 6$
(ii) $-2 \times -3 \times -4 \times -5 = 120$
(iii) $-2 \times -3 \times -4 \times -5 \times -6 \times -7 = 5040$

In (i), given above, the product of two negative integers gives a positive integer.
In (ii), given above, the product of four negative integers gives a positive integer.
In (iii), given above, the product of *six* negative integers gives a positive integer.
Thus, we conclude that *the proudct of even number of negative integers is always positive.*

1. $(-a_1) \times (-a_2) \times (-a_3) \times (-a_4) \times (-a_5) \times$ .... upto an odd number of negative integers
$= -(a_1 \times a_2 \times a_3 \times a_4 \times a_5 \times ....)$
= a negative integer
2. $(-a_1) \times (-a_2) \times (-a_3) \times (-a_4) \times$ .... upto an even number of negative integers
$= a_1 \times a_2 \times a_3 \times a_4 \times$ ....
= a positive integer.

Example 1 :

Evaluate :

(i) $23 \times -3 + (-23) \times 97$ (ii) $897 \times 99 + 897$ (iii) $1389 \times 450 - 389 \times 450$

Solution :

(i) $23 \times -3 + (-23) \times 97$
$= (-23) \times 3 + (-23) \times 97$
$= (-23) \times (3 + 97) = -23 \times 100 = -2300$ (Ans.)

(ii) $897 \times 99 + 897$
$= 897 \times 99 + 897 \times 1$
$= 897 \times (99 + 1) = 897 \times 100 = 89700$ (Ans.)

(iii) $1389 \times 450 - 389 \times 450$
$= (1389 - 389) \times 450$
$= 1000 \times 450 = 450000$ (Ans.)

Example 2 :

Evaluate :

(i) $3 \times 5 \times 6$
(ii) $3 \times (-5) \times 6$
(iii) $3 \times (-5) \times (-6)$
(iv) $(-3) \times (-5) \times (-6)$

Solution :

(i) $3 \times 5 \times 6 = 90$ (Ans.)

(ii) $3 \times -5 \times 6$ has only one negative integer
$\therefore 3 \times -5 \times 6 = -90$ (Ans.)

(iii) $3 \times (-5) \times (-6)$ has two negative integers and the product of two negative integers is always positive, therefore :
$\therefore 3 \times (-5) \times (-6) = 3 \times 30 = 90$ (Ans.)

(iv) $(-3) \times (-5) \times (-6)$ has three negative integers and the product of three negative integers is always negative.
$\therefore (-3) \times (-5) \times (-6) = -(3 \times 5 \times 6) = -90$ (Ans.)

Example 3 :

Evaluate :

(i) (–2) × (–4) × (–6) × (–5)

(ii) (–1) × (–4) × (–5) × (–7) × (–8)

(iii) (–1) × (–1) × (–1) × ...... 20 times

(iv) (–1) × (–1) × (–1) × ...... 25 times

Solution :

(i) Since, the number of negative integers in the product is even, therefore the result of this product will be positive.

Hence, (–2) × (–4) × (–6) × (–5) = 2 × 4 × 6 × 5 = 240 **(Ans.)**

(ii) Since, the number of negative integers in the product is odd, therefore the result of this product will be negative.

Hence, (–1) × (–4) × (–5) × (–7) × (–8) = –(1 × 4 × 5 × 7 × 8) = –1120 **(Ans.)**

(iii) Since, the number of negative integers in the product is even, therefore the result of this product will be positive.

Hence, (–1) × (–1) × (–1) × (–1) × ...... 20 times

= 1 × 1 × 1 × 1 × ...... 20 times = 1 **(Ans.)**

(iv) Since, the number of negative integers in the product is odd, therefore the result of this product will be negative.

Hence, (–1) × (–1) × (–1) × (–1) × ...... 25 times

= – (1 × 1 × 1 × 1 × ...... 25 times) = –1 **(Ans.)**

Example 4 :

Complete the adjoining multiplication table :

Is the multiplication table symmetrical about the diagonal joining the upper left corner to the lower right corner ?

X	–3	–2	–1	0	1	2	3
–3							
–2							
–1							
0							
1							
2							
3							

Solution :

The required multiplication table will be as given below :

A

X	–3	–2	–1	0	1	2	3
–3	9	6	3	0	–3	–6	–9
–2	6	4	2	0	–2	–4	–6
–1	3	2	1	0	–1	–2	–3
0	0	0	0	0	0	0	0
1	–3	–2	–1	0	1	2	3
2	–6	–4	–2	0	2	4	6
3	–9	–6	–3	0	3	6	9

B

Yes, the multipication table is symmetric about the diagonal joining the upper left corner to the lower right corner.

AB is the diagonal joining the upper left corner to the lower right corner.

EXERCISE 1(A)

1. Evaluate :
   (i) $427 \times 8 + 2 \times 427$ (ii) $394 \times 12 + 394 \times (-2)$ (iii) $558 \times 27 + 3 \times 558$
2. Evaluate :
   (i) $673 \times 9 + 673$ (ii) $1925 \times 101 - 1925$
3. Verify :
   (i) $37 \times \{8 + (-3)\} = 37 \times 8 + 37 \times (-3)$
   (ii) $(-82) \times \{(-4) + 19\} = (-82) \times (-4) + (-82) \times 19$
   (iii) $\{7 - (-7)\} \times 7 = 7 \times 7 - (-7) \times 7$
   (iv) $\{(-15) - 8\} \times -6 = (-15) \times (-6) - 8 \times (-6)$
4. Evaluate :
   (i) 15×8 (ii) $15 \times (-8)$
   (iii) $(-15) \times 8$ (iv) $(-15) \times - 8$
5. Evaluate :
   (i) $4 \times 6 \times 8$ (ii) $4 \times 6 \times (-8)$ (iii) $4 \times (-6) \times 8$
   (iv) $(-4) \times 6 \times 8$ (v) $4 \times (-6) \times (-8)$ (vi) $(-4) \times (-6) \times 8$
   (vii) $(-4) \times 6 \times (-8)$ (viii) $(-4) \times (-6) \times (-8)$
6. Evaluate :
   (i) $2 \times 4 \times 6 \times 8$ (ii) $2 \times (-4) \times 6 \times 8$ (iii) $(-2) \times 4 \times (-6) \times 8$
   (iv) $(-2) \times (-4) \times 6 \times (-8)$ (v) $(-2) \times (-4) \times (-6) \times (-8)$
7. Determine the integer whose product with '–1' is :
   (i) –47 (ii) 63
   (iii) –1 (iv) 0
8. Eighteen integers are multiplied together. What will be the sign of their product, if :
   (i) 15 of them are negative and 3 are positive ?
   (ii) 12 of them are negative and 6 are positive ?
   (iii) 9 of them are positive and the remaining are negative ?
   (iv) all are negative ?
9. Find which is greater ?
   (i) $(8 + 10) \times 15$ or $8 + 10 \times 15$ (ii) $12 \times (6 - 8)$ or $12 \times 6 - 8$
   (iii) $\{(-3) - 4\} \times (-5)$ or $(-3) - 4 \times (-5)$
10. State, **true** or **false** :
    (i) product of two different integers can be zero.
    (ii) product of 120 negative integers and 121 positive integers is negative.
    (iii) $a \times (b + c) = a \times b + c$
    (iv) $(b - c) \times a = b - c \times a$.

1.4 DIVISION OF INTEGERS :

Division is an inverse process of multiplication.

To divide 36 by 9 means to find an integer which on multiplying with 9 gives 36. Such an integer is 4. This fact is expressed as :

$36 \div 9 = 4 \Rightarrow 4 \times 9 = 36$

Similarly, $75 \div 15 = 5 \Rightarrow 5 \times 15 = 75$

Remember :

1. *Division of an integer by an integer of same sign is always positive.*

   i.e. $\dfrac{\text{one integer}}{\text{an integer with same sign}}$ = a positive number.

For example :

(i) $\dfrac{-24}{-6} = 4$ (ii) $\dfrac{24}{6} = 4$

(iii) $\dfrac{32}{20} = \dfrac{8 \times 4}{5 \times 4} = \dfrac{8}{5}$ (iv) $\dfrac{-38}{-57} = \dfrac{-(2 \times 19)}{-(3 \times 19)} = \dfrac{2}{3}$ and so on

$\dfrac{+\text{ve integer}}{+\text{ve integer}}$ = a positive number

$\dfrac{-\text{ve integer}}{-\text{ve integer}}$ = a positive number

2. *Division of an integer by an integer of opposite sign is always negative.*

   i.e. $\dfrac{\text{one integer}}{\text{an integer with opposite sign}}$ = a negative number.

For example :

(i) $\dfrac{-24}{6} = -4$ (ii) $\dfrac{24}{-6} = -4$

(iii) $\dfrac{32}{-20} = \dfrac{8 \times 4}{-5 \times 4} = -\dfrac{8}{5}$ (iv) $\dfrac{-38}{57} = \dfrac{-2 \times 19}{3 \times 19} = -\dfrac{2}{3}$

$\dfrac{+\text{ve integer}}{-\text{ve integer}}$ = a negative number

$\dfrac{-\text{ve integer}}{+\text{ve integer}}$ = a negative number

Note 1 : In a division,

- the number to be divided is called **dividend**, thus in $65 \div 13 = 5$; dividend = 65.
- the number which divides is called **divisor**; thus in $65 \div 13 = 5$, divisor = 13.
- the result of division is called **quotient**; thus in $65 \div 13 = 5$, quotient = 5.

Note 2 : Dividend ÷ divisor = quotent *i.e.* $\dfrac{\text{dividend}}{\text{divisor}}$ = quotient

- if dividend is positive and divisor is negative, the quotient is negative.
- if dividend is negative and divisor is positive, the quotient is negative.
- if dividend and divisor both are positive or both are negative, the quotient is positive.

The following table will make the above concept more clear :

Dividend	Divisor	Dividend ÷ Divisor	Quotient
40	8	$40 \div 8 = \frac{40}{8}$	5
–40	8	$(-40) \div 8 = \frac{-40}{8}$	–5
–40	–8	$(-40) \div (-8) = \frac{-40}{-8}$	5
40	–8	$40 \div -8 = \frac{40}{-8}$	–5
Remember : $\frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$			

1.5 PROPERTIES OF DIVISION OF INTEGERS :

1. *If m and n are two integers, then m ÷ n is not necessarily an integer.*

e.g. $18 \div 5 = \frac{18}{5}$, *which is not an integer.*

$-14 \div 4 = \frac{-14}{4} = -\frac{7}{2}$, *which is not an integer.*

2. *If m is a non-zero integer then m ÷ m = 1.*

e.g. $12 \div 12 = \frac{12}{12} = 1$

$17 \div 17 = \frac{17}{17} = 1, (-25) \div (-25) = \frac{-25}{-25} = 1.$

3. *For every non-zero integer m, m ÷ 1 = m.*

e.g. $8 \div 1 = \frac{8}{1} = 8, (-16) \div 1 = \frac{-16}{1} = -16$

$1 \div 1 = \frac{1}{1} = 1$

4. *For every non-zero integer m, 0 ÷ m = 0.*

e.g. $0 \div 6 = \frac{0}{6} = 0,\ 0 \div (-11) = \frac{0}{-11} = 0$

$0 \div 1 = \frac{0}{1} = 0$

5. *For every integer m, m ÷ 0 is not meaningful* (not defined).

e.g. $8 \div 0 = \frac{8}{0}$, *which is not meaningful*

$(-23) \div 0 = \dfrac{-23}{0}$, *which is not meaningful*

6. *If l, m and n are non-zero integers, then*
   $(l \div m) \div n \neq l \div (m \div n)$ unless $n = 1$.

When $n = 1$, $(l \div m) \div n = (l \div m) \div 1 = l \div m$
and, $l \div (m \div n) = l \div (m \div 1) = l \div m$

7. If l, m and n are integers, then
   (i) $l > m$ and n is positive $\Rightarrow (l \div n) > (m \div n)$
   e.g. $15 > 6$ and 3 is positive $\Rightarrow (15 \div 3) > (6 \div 3)$ *i.e.* $5 > 2$
   (ii) $l >$ m and n is negative $\Rightarrow (l \div n) < (m \div n)$
   e.g. $18 > 9$ and -3 is negative $\Rightarrow \{18 \div (-3)\} < \{9 \div (-3)\}$

$$\Rightarrow \frac{18}{-3} < \frac{9}{-3} \quad \text{i.e. } -6 < -3$$

An integer is negative means it is less than 0.
$(-5) < 0$, $(-12) < 0$, $(-23) < 0$ and so on.
An integer is positive means it is greater then 0.
$5 > 0$, $12 > 0$, $23 > 0$ and so on.

Example 5 :

Divide :

(i) 132 by 11
(ii) (-132) by 11
(iii) 132 by (-11)
(iv) (-132) by (-11)

Solution :

(i) $132 \div 11 = \dfrac{132}{11} = \dfrac{12 \times 11}{11} = 12$ **(Ans.)**

(ii) $(-132) \div 11 = \dfrac{-132}{11} = \dfrac{-12 \times 11}{11} = -12$ **(Ans.)**

(iii) $132 \div (-11) = \dfrac{132}{-11}$

$= -\dfrac{132}{11}$ $\left[\because \dfrac{a}{-b} = \dfrac{-a}{b} = -\dfrac{a}{b}\right]$

$= -\dfrac{12 \times 11}{11} = -12$ **(Ans.)**

(iv) $(-132) \div (-11) = \dfrac{-132}{-11} = \dfrac{132}{11} = \dfrac{12 \times 11}{11} = 12$ **(Ans.)**

Example 6 :

Evaluate :

(i) $(-91) \div 13$ (ii) $-98 \div 7$ (iii) $162 \div (-27)$

Solution :

(i) $(-91) \div 13 = \dfrac{-91}{13} = \dfrac{-7 \times 13}{13} = -7$ **(Ans.)**

(ii) $-98 \div 7 = \dfrac{-98}{7} = \dfrac{-14 \times 7}{7} = -14$ **(Ans.)**

(iii) $162 \div (-27) = \dfrac{162}{-27} = \dfrac{-162}{27} = \dfrac{-6 \times 27}{27} = -6$ **(Ans.)**

Example 7 :

Find the quotient in each of the following :

(i) $1872 \div 12$ (ii) $3159 \div (-13)$ (iii) $-17000 \div 125$

(iv) $-1352 \div (-26)$

Solution :

(i) **Required quotient** $= \dfrac{1872}{12} = \dfrac{156 \times 12}{12} = 156$ **(Ans.)**

(ii) **Required quotient** $= \dfrac{3159}{-13} = -\dfrac{3159}{13} = -\dfrac{243 \times 13}{13} = -243$ **(Ans.)**

(iii) **Required quotient** $= \dfrac{-17000}{125} = \dfrac{-136 \times 125}{125} = -136$ **(Ans.)**

(iv) **Required quotient** $= \dfrac{-1352}{-26} = \dfrac{1352}{26} = 52$ **(Ans.)**

1.6 USING DMAS

In DMAS,

D stands for **division**
M stands for **multiplication**
A stands for **addition**
and **S** stands for **subtraction.**

1. **DAMS** represents four fundamental operations **D** (division), **M** (multiplication), **A** (addition) and **S** (subtraction).
2. If an expression has more than one fundamental operations, we perfrom operations using the rule of DMAS *i.e.* **first of all** we perfrom **D** (division), **then M** (multiplication), **then A** (addition) and **in the last S** (subtraction).

Example 8 :

Evaluate : 18 – 6 ÷ 3 × 4.

Solution :

18 – 6 ÷ 3 × 4

= 18 – 2 × 4

= 18 – 8

= 10 **(Ans.)**

Using DMAS

Division (D) : 6 ÷ 3 = 2

Multiplication (M) : 2 × 4 = 8

Subtraction (S) : 18 – 8 = 10

Example 9 :

Evaluate : (–10) + (–4) ÷ (–2) × 3

Solution :

(–10) + (–4) ÷ (–2) × 3

= –10 + 2 × 3

= –10 + 6

= –4 **(Ans.)**

Using DMAS

Division (D) : (–4) ÷ (–2) = 2

Multiplication (M) : 2 × 3 = 6

Addition (A) : –10 + 6 = –4

Example 10 :

Evaluate : 5 + (–48) ÷ (–16) + (–2) × 6

Solution :

5 + (–48) ÷ (–16) + (–2) × 6

= 5 + 3 + (–2) × 6

= 5 + 3 + (–12)

= 8 – 12

= –4 **(Ans.)**

Using DMAS

Division (D) : (–48) ÷ (–16) = 3

Multiplication (M) : (–2) × 6 = –12

Addition (A) : 5 + 3 = 8

Subtraction (S) : 8 – 12 = –4

EXERCISE 1(B)

1. Divide :
   - (i) 117 by 9
   - (ii) (–117) by 9
   - (iii) 117 by (–9)
   - (iv) (–117) by (–9)
   - (v) 225 by (–15)
   - (vi) (–552) ÷ 24
   - (vii) (–798) by (–21)
   - (viii) (–910) ÷ 26
2. Evaluate :
   - (i) (–234) ÷ 13
   - (ii) 234 ÷ (–13)
   - (iii) (–234) ÷ (–13)
   - (iv) 374 ÷ (–17)
   - (v) (–374) ÷ 17
   - (vi) (–374) ÷ (–17)
   - (vii) (–728) ÷ 14
   - (viii) 272 ÷ (–17)
3. Find the quotient in each of the following divisions :
   - (i) 299 ÷ 23
   - (ii) 299 ÷ (–23)
   - (iii) (–384) ÷ 16
   - (iv) (–572) ÷ (–22)
   - (v) 408 ÷ (–17)
4. Divide :
   - (i) 204 by 17
   - (ii) 152 by – 19
   - (iii) 0 by 35
   - (iv) 0 by (–82)
   - (v) 5490 by 10
   - (vi) 762800 by 100

5. State, true or false :
   (i) $0 \div 32 = 0$
   (ii) $0 \div (-9) = 0$
   (iii) $(-37) \div 0 = 0$
   (iv) $0 \div 0 = 0$

6. Evaluate :
   (i) $42 \div 7 + 4$
   (ii) $12 + 18 \div 3$
   (iii) $19 - 20 \div 4$
   (iv) $16 - 5 \times 3 + 4$
   (v) $6 - 8 - (-6) \div 2$
   (vi) $13 - 12 \div 4 \times 2$
   (vii) $16 + 8 \div 4 - 2 \times 3$
   (viii) $16 \div 8 + 4 - 2 \times 3$
   (ix) $16 - 8 + 4 \div 2 \times 3$
   (x) $(-4) + (-12) \div (-6)$
   (xi) $(-18) + 6 \div 3 + 5$
   (xii) $(-20) \times (-1) + 14 \div 7$

1.7 REMOVAL OF BRACKETS

In removal of brackets, operations within the bracket are performed before the operations outside it.

For example :

(i)
$$(24 \div 3) \times 2 = \frac{24}{3} \times 2$$
$$= 8 \times 2 = 16 \qquad [\because\ (24 \div 3) = 8]$$

(ii)
$$24 \div (3 \times 2) = 24 \div 6$$
$$= \frac{24}{6} = 4 \qquad [\because\ 24 \div 6 = 4]$$

(iii)
$$18 \div (2 \times 18 \div 6 - 5) = 18 \div (2 \times 3 - 5) \qquad [18 \div 6 = 3]$$
$$= 18 \div (6 - 5) \qquad [2 \times 3 = 6]$$
$$= 18 \div 1 \qquad [6 - 5 = 1]$$
$$= \frac{18}{1} = 18$$

Important : Whenever a number is written just before a bracket, with numbers inside the bracket and separated by **plus** or **minus** signs; the bracket is removed and at the same time each number inside the bracket is multiplied by the number written outside it.

For example :

(i) $3(a - b) = 3a - 3b$

(ii) $5(3a + 2b) = 15a + 10b$

(iii) $2(5 + 4 - 3) = 10 + 8 - 6 = 18 - 6 = 12$

(iv) $5(3 - 8 + 10) = 15 - 40 + 50 = 65 - 40 = 25$

(v) $-4(a + b - 3) = -4a - 4b + 12$

(vi) $-3(15 - 18 + 2) = -3(17 - 18) = -3 \times -1 = 3$

In a complex expression, many types of brackets are used. The most commonly used brackets are :

	Bracket's symbol	**Bracket's name**
1.	( )	Small bracket or Parentheses
2.	{ }	Curly (middle) bracket
3.	[ ]	Square bracket
4.	—	Vinculum.

Note : If in an expression, all the four types of brackets, as discussed above, are used, first of all we simplify expression below vinuculum, then inside small bracket, then inside curly bracket and lastly inside square bracket.

Example 11 :

Evaluate : $30 - [26 - \{15 + (8 - \overline{6 - 3})\}]$.

Solution :

$30 - [26 - \{15 + (8 - \overline{6 - 3})\}]$

$= 30 - [26 - \{15 + (8 - 3)\}]$ [Removing vinculum]

$= 30 - [26 - \{15 + 5\}]$ [Removing small bracket]

$= 30 - [26 - 20]$ [Removing curly bracket]

$= 30 - 6$ [Removing square bracket]

$= 24$ **(Ans.)**

Order in which the brackets must be removed :

1. **When brackets used are :** $[\{(\overline{\quad})\}]$
   order of removing the brackets is : firstly '—', then ( ), then { } and finally [ ].
2. **When brackets used are :** [{( )}]
   order of removing the brackets is : firstly ( ), then { } and finally [ ].
3. **When brackets used are :** {( )}
   order of removing the brackets is : firstly ( ) and finally { }.

1. If there is a **plus sign** before a bracket, the bracket is removed without changing the sign of any term inside the bracket.
   i.e. (i) $+ (a - b + c) = a - b + c$
   (ii) $+ 5\,(2a + b - 3c) = 10a + 5b - 15c$
2. If there is a **minus sign** before a bracket, the bracket is removed by changing the sign of each term inside the bracket.
   i.e. (i) $- (a - b + c) = - a + b - c$
   (ii) $- 5\,(2a + b - 3c) = - 10a - 5b + 15c$

Example 12 :

Evaluate : $30 - [8 + \{31 - (32 - 10)\}]$.

Solution :

$30 - [8 + \{31 - (32 - 10)\}]$

$= 30 - [8 + \{31 - 22\}]$

$= 30 - [8 + 9] = 30 - 17 = 13$ (Ans.)

Example 13 :

Evaluate : $55 - \left[25 - \{23 - (12 - \overline{11 - 8})\}\right]$

Solution :

$55 - \left[25 - \{23 - (12 - \overline{11 - 8})\}\right]$

$= 55 - [25 - \{23 - (12 - 3)\}]$

$= 55 - [25 - \{23 - 9\}]$

$= 55 - [25 - 14] = 55 - 11 = 44$ (Ans.)

Example 14 :

Evaluate : $45 - \left[28 - \{34 - (24 - \overline{14 - 8})\}\right]$

Solution :

$45 - \left[28 - \{34 - (24 - \overline{14 - 8})\}\right]$

$= 45 - [28 - \{34 - (24 - 6)\}]$

$= 45 - [28 - \{34 - 18\}]$

$= 45 - [28 - 16] = 45 - 12 = 33$ (Ans.)

Example 15 :

Evaluate : $-\{4 - (5 - 2)\}$.

Solution :

$-\{4 - (5 - 2)\}$

$= -\{4 - 5 + 2\}$ $[-(5 - 2) = -5 + 2]$

$= -4 + 5 - 2 = -6 + 5 = -1$ (Ans.)

Alternative method :

$= -\{4 - (5 - 2)\} = -\{4 - 3\} = -1$ (Ans.)

Example 16 :

Evaluate : $3\{17 - 4(8 - 6)\}$.

Solution :

$3\{17 - 4(8 - 6)\}$

$= 3\{17 - 32 + 24\}$

$= 51 - 96 + 72 = 123 - 96 = 27$ $[-4(8 - 6) = -4 \times 8 + 4 \times 6 = -32 + 24]$ (Ans.)

Alternative method :

$$3\{17-4(8-6)\} = 3\{17-4\times 2\}$$
$$= 3\{17-8\} = 3\times 9 = 27$$

(Ans.)

EXERCISE 1(C)

Evaluate :

1. $18-(20-15\div 3)$.
2. $-15+24\div(15-13)$.
3. $35-\{15+14-(13+\overline{2-1+3})\}$
4. $27-\{13+4-(8+4-\overline{1+3})\}$
5. $32-[43-\{51-(20-\overline{18-7})\}]$
6. $46-[26-\{14-(15-4\div 2\times 2)\}]$
7. $45-[38-\{60\div 3-(6-9\div 3)\div 3\}]$
8. $17-[17-\{17-(17-\overline{17-17})\}]$
9. $2550-[510-\{270-(90-\overline{80+7})\}]$
10. $30+[\{-2\times(25-\overline{13-3})\}]$
11. $88-\{5-(-48)\div(-16)\}$
12. $9\times(8-\overline{3+2})-2(2+\overline{3+3})$
13. $2-[3-\{6-(5-\overline{4-3})\}]$

EXERCISE 1(D)

1. The sum of two integers is –15. If one of them is 9, find the other.
2. The difference between integers x and –6 is –5. Find the values of x.

   $x-(-6)=-5$ or $-6-x=-5$

3. The sum of two integers is 28. If one intger is –45, find the other.
4. The sum of two integers is –56. If one integer is –42, find the other.
5. The difference between an integer x and (–9) is 6. Find all possible values of x.
6. Evaluate :
   (i) $(-1)\times(-1)\times(-1)\times$ .... 60 times.
   (ii) $(-1)\times(-1)\times(-1)\times(-1)\times$ .... 75 times.
7. Evaluate :
   (i) $(-2)\times(-3)\times(-4)\times(-5)\times(-6)$
   (ii) $(-3)\times(-6)\times(-9)\times(-12)$
   (iii) $(-11)\times(-15)+(-11)\times(-25)$
   (iv) $10\times(-12)+5\times(-12)$
8. (i) If $x\times(-1)=-36$, is x positive or negative ?
   (ii) If $x\times(-1)=36$, is x positive or negative ?

9. Write all the integers between –15 and 15, which are divisible by 2 and 3.

10. Write all the integers between –5 and 5, which are divisible by 2 or 3.

11. Evaluate :

(i) $(-20) + (-8) \div (-2) \times 3$

(ii) $(-5) - (-48) \div (-16) + (-2) \times 6$

(iii) $16 + 8 \div 4 - 2 \times 3$

(iv) $16 \div 8 \times 4 - 2 \times 3$

(v) $27 - [5 + \{28 - (29 - 7)\}]$

(vi) $48 - [18 - \{16 - (5 - \overline{4 + 1})\}]$

(vii) $-8 - \{-6\,(9 - 11) + 18 \div -3\}$

(viii) $(24 \div \overline{12 - 9} - 12) - (3 \times 8 \div 4 + 1)$

12. Find the result of subtracting the sum of all integers between 20 and 30 from the sum of all integers from 20 to 30.

13. Add the product of (–13) and (–17) to the quotient of (–187) and 11.

14. The product of two integers is –180. If one of them is 12, find the other.

15. (i) A number changes from –20 to 30. What is the increase or decrease in the number ?

(ii) A number changes from 40 to –30. What is the increase or decrease in the number ?

RATIONAL NUMBERS 2

2.1 INTRODUCTION

If a and b are two integers, then each of $a + b$, $a - b$ and $a \times b$ is also an integer. However, it is not necessary that $a \div b$ or $b \div a$ is also an integer.

For example :

Consider two integers 8 and 5. Clearly :

(i) $8 + 5 = 13$ is an integer.

(ii) $8 - 5 = 3$ is an integer

$5 - 8 = -3$ is also an integer.

(iii) $8 \times 5 = 40$ is an integer

But $8 \div 5 = \frac{8}{5}$ is not an integer and

$5 \div 8 = \frac{5}{8}$ is also not an integer.

Here, $\frac{5}{8}$ and $\frac{8}{5}$ are fractions and are said to form the system of rational numbers.

2.2 RATIONAL NUMBERS

If p is an integer, q is an integer and $q \neq 0$, then the number of the form $\frac{p}{q}$ is called a rational number.

Infact, the number having a value that can be expressed as the result of dividing an integer by a non-zero integer, is defined as rational number.

For example :

(i) $\frac{3}{7}$ is a rational number as 3 and 7 both are integers and $7 \neq 0$.

(ii) $-\frac{15}{19}$ is a rational number as -15 is an integer, 19 is an integer and $19 \neq 0$.

In the same way, each of the following numbers is a rational number.

$\frac{3}{4}, \frac{-4}{7}, \frac{-12}{-5}, \frac{8}{-9}$, ...

Remember :

1. *Every integer is a rational number but the converse is not ture.*

   (i) $5 = \frac{5}{1}$ is a rational number as 5 and 1 are integers and $1 \neq 0$.

   (ii) $-8 = \frac{-8}{1}$ is a rational number as -8 and 1 are integers and $1 \neq 0$.

   (iii) $0 = \frac{0}{1}$ is a rational number as 0 and 1 are integers and $1 \neq 0$.

1. $\frac{3}{5}$ is a rational number as 3 and 5 are integers and $5 \neq 0$, but $\frac{3}{5}$ is not an integer.

Every natural number is an integer and so a rational number.

Every whole number is also an integer and so a rational number.

2. $0 = \frac{0}{1}$, $0 = \frac{0}{5}$, $0 = \frac{0}{-8}$, etc.

Zero is a rational number.

2. *Every fraction is a rational number but the converse is not rue.*
3. In the rational number in the form of a fraction $\frac{p}{q}$, p is called the numerator and q is called the denominator.
   (i) In $\frac{7}{11}$, numerator = 7 and denominator = 11.
   (ii) $\because 13 = \frac{13}{1}$, numerator = 13 and denominator = 1.
4. *Positive rational number :* A rational number is said to be positive, if its numerator and denominator both are either positive or negative.
5. *Negative rational number :* A rational number is said to be negative, if its numerator and denominator are with opposite signs *i.e.* if numerator is positive, denominator is negative and if numerator is negative, denominator is positive.

   Thus each of $\frac{-5}{7}$, $\frac{8}{-11}$, $\frac{15}{-13}$, $\frac{-23}{25}$, etc. is a negative rational number.

Remember : (i) $\frac{-5}{7} = \frac{5}{-7} = -\frac{5}{7}$

(ii) $\frac{8}{-11} = \frac{-8}{11} = -\frac{8}{11}$

6. (i) *Every positive integer (i.e. every natural number) is a positive rational number.*

   e.g. For natural number $5 = \frac{5}{1}$ is a positive rational number.

   Similarly, $10 = \frac{10}{1}$ is a positive rational number.

   (ii) *Every negative integer is a negative rational number.*

   e.g. $-5 = \frac{-5}{1}$ which is a negative rational number.

   In the same way, each of $-12 = \frac{-12}{1}$, $-28 = -\frac{28}{1}$, ... is a negative rational number.

Zero (0) is a rational number but it is neither positive nor negative

EXERCISE 2(A)

1. Write down a rational number whose numerator is the largest number of two digits and denominator is the smallest number of four digits.

2. Write the numerator of each of the following rational numbers :

   (i) $\frac{-125}{127}$ (ii) $\frac{37}{-137}$ (iii) $\frac{-85}{93}$

   (iv) 2 (v) 0

3. Write the denominator of each of the following rational numbers :

   (i) $\frac{7}{-15}$ (ii) $\frac{-18}{29}$ (iii) $\frac{-3}{4}$

   (iv) -7 (v) 0

4. Write down a rational number with numerator $(-5) \times (-4)$ and with denominator $(28 - 27) \times (8 - 5)$.

5. (i) $\frac{-15}{1}$ in integer form is .... (ii) $\frac{23}{-1}$ in integer form is ....

   (iii) If $18 = \frac{18}{a}$ then a = ... (iv) If $-57 = \frac{57}{a}$ then a = ...

6. Separate positive and negative rational numbers from the following :

   $\frac{-3}{5}, \frac{3}{-5}, \frac{-3}{-5}, \frac{3}{5}, 0, \frac{-13}{-3}, \frac{15}{-8}, \frac{-15}{8}$

7. Find three rational numbers equivalent to

   (i) $\frac{3}{5}$ (ii) $\frac{4}{-7}$ (iii) $\frac{-5}{9}$ (iv) $\frac{8}{-15}$

   (i) $\frac{4}{-7} = \frac{4\times 2}{-7\times 2} = \mathbf{\frac{8}{-14}}, \frac{4\times 3}{-7\times 3} = \mathbf{\frac{12}{-21}}$ and $\frac{4\times 4}{-7\times 4} = \mathbf{\frac{16}{-28}}$

   $\Rightarrow$ Rational numbers $\frac{8}{-14}, \frac{12}{-21}$ and $\frac{16}{-28}$ are equivalent to the given rational number $\frac{4}{-7}$.

8. Which of the following are not rational numbers :

   (i) -3 (ii) 0 (iii) $\frac{0}{4}$

   (iv) $\frac{8}{0}$ (v) $\frac{0}{0}$

9. Express each of the following integers as a rational number with denominator 7 :

   (i) 5 (ii) -8 (iii) 0

   (iv) -16 (v) 7

   (ii) $-8 = \frac{-8\times 7}{7} = -\frac{56}{7}$ and (iii) $0 = \frac{0\times 7}{7} = \frac{0}{7}$

10. Express $\frac{3}{5}$ as a rational number with denominator :

(i) 20 (ii) −20 (iii) 45 (iv) 25 (v) −35

(i) $\frac{3}{5} \times \frac{4}{4} = \frac{12}{20}$ (v) $\frac{3}{5} \times \frac{-7}{-7} = \frac{-21}{-35}$

11. Express $\frac{4}{7}$ as a rational number with numerator :

(i) 12 (ii) −12 (iii) −16 (iv) −20 (v) 20

(i) $\frac{4}{7} = \frac{4}{7} \times \frac{3}{3} = \mathbf{\frac{12}{21}}$ (iii) $\frac{4}{7} = \frac{4 \times -4}{7 \times (-4)} = \mathbf{\frac{-16}{-28}}$

12. Find *x*, such that :

(i) $-\frac{2}{3} = \frac{6}{x}$ (ii) $\frac{7}{-4} = \frac{x}{8}$ (iii) $\frac{3}{7} = \frac{x}{-35}$

(iv) $\frac{-48}{x} = 6$ (v) $\frac{36}{x} = 3$ (vi) $\frac{-27}{x} = 9$

$$\frac{3}{2} = \frac{x}{14} \Rightarrow \frac{3 \times 7}{2 \times 7} = \frac{x}{14} \Rightarrow \frac{21}{14} = \frac{x}{14} \Rightarrow x = 21$$

$$\frac{-5}{3} = \frac{-15}{x} \Rightarrow \frac{-5}{3} \times \frac{3}{3} = \frac{-15}{x} \Rightarrow \frac{-15}{9} = \frac{-15}{x} \Rightarrow x = 9$$

13. Express each of the following rational numbers to the lowest terms :

(i) $\frac{12}{15}$ (ii) $\frac{-120}{144}$ (iii) $\frac{-48}{-72}$ (iv) $\frac{14}{-56}$

(iv) $\mathbf{\frac{14}{-56}} = \frac{2 \times 7}{-2 \times 2 \times 2 \times 7}$

$= \frac{1}{-4} = -\mathbf{\frac{1}{4}}$

2	14
	7

2	56
2	28
2	14
	7

Alternative method :

Find H.C.F. of numerator 14 and denominator 56 which is 14.

Now divide each term of the given rational number by their H.C.F. 14 (obtained above) to get the required answer.

Clearly, $\mathbf{\frac{14}{-56}} = \frac{14 \div 14}{-56 \div 14} = \frac{1}{-4} = -\mathbf{\frac{1}{4}}$

14. Express each of the following rational numbers in the standard form.

(i) $\frac{-7}{-8}$ (ii) $\frac{5}{-12}$ (iii) $\frac{-7}{-20}$ (iv) $\frac{4}{-9}$

A rational number is said to be in standard form, if its denominator is positive in lowest term.

2.3 REPRESENTATION OF RATIONAL NUMBERS ON NUMBER LINE

1. Draw a straight line. Mark a point O on this line and name it 0 (zero).
2. On the right side of O, mark points A, B, C, D, E, ... on the line drawn such that the distance between consecutive points is same

   i.e. OA = AB = BC = CD = ...
3. In the same way, on the same line mark points A′, B′, C′, D′, etc. on the left side of O such that :

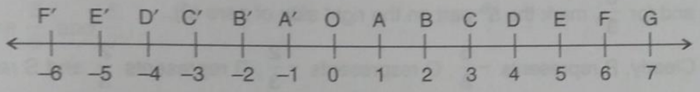
   OA′ = A′B′ = B′C′ = ...

   On the whole, we must have ;

   .... B′C′ = A′B′ = OA′ = OA = AB = BC = CD = ...

   Clearly, if the point A, B, C, D, E, ..., which are on the right side of zero (0); represent positive integers 1, 2, 3, 4, 5, ..., the points A′, B′, C′, D′, E′, ... will represent negative integers −1, −2, −3, −4, −5, ...

   See the number line, given below :

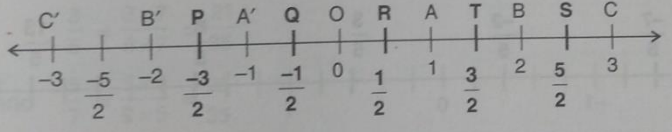


Example 1 :

Represent rational numbers $-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$ and $\frac{5}{2}$ on the same number line :

Solution :

Draw a number line as shown below :



Consider C′B′ = B′A′ = A′O = OA = AB = BC = one unit length

Clearly, A represents 1, B represents 2 and C represents 3.

In the same way, A′ represents −1, B′ represents = −2 and C′ represents −3.

Since, denominator of each given rational number is 2, mark middle points of each of OA, AB, BC and OA′, A′B′, B′C′ to divide each of OA, AB, BC and OA′, A′B′, B′C′ into two equal parts. Now the given points are marked on this number line.

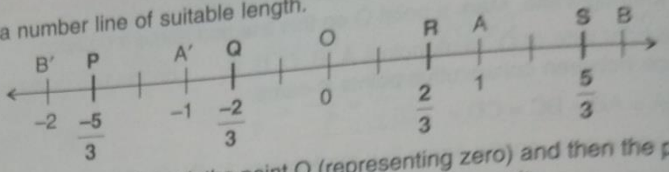
In the number line now obtained, P represents $-\frac{3}{2}$, Q respresents $-\frac{1}{2}$, R represents $\frac{1}{2}$, T represents $\frac{3}{2}$ and S represents $\frac{5}{2}$.

Example 2 :

Represent rational numbers $-\frac{5}{3}, -\frac{2}{3}, \frac{2}{3}$ and $\frac{5}{3}$ on the same number line :

Solution :

Draw a number line of suitable length.



On this line, first of all mark the point O (representing zero) and then the points A, B, B′, etc., such that OA = AB = OA′ = A′B′ = ..., = one unit.

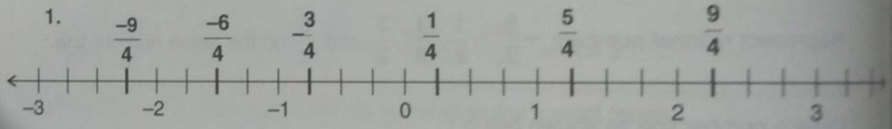
Since, the denominator of each given rational number is 3, divide each of OA, AB, A′B′, etc. into three equal parts. Now, each smaller part so obtained is equal to $\frac{1}{3}$.

For $-\frac{5}{3}$, mark the point P at the 5th part on the left side of 0, for $-\frac{2}{3}$, mark the poin at the 2nd part on the left side of 0, for $\frac{2}{3}$ mark R at the 2nd part on the right side of and for $\frac{5}{3}$, mark the 5th part on the right side of zero (0)..

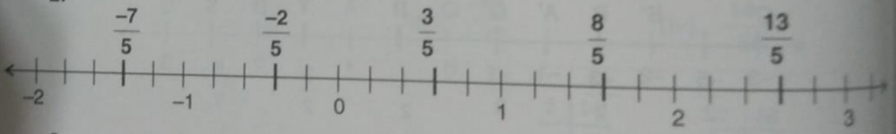
Clearly, P represents $-\frac{5}{3}$, Q respresents $-\frac{2}{3}$, R represents $\frac{2}{3}$ and S represents $\frac{5}{3}$

The following number lines will make the concept more clear :

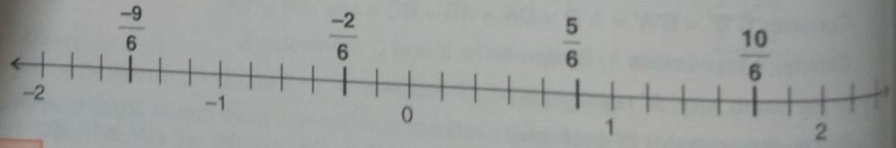
1.



2.



3.



2.4 COMPARING RATIONAL NUMBERS

1. Every positive rational number is greater than 0 (zero) and is greater than every negative number :
   e.g. 8 > 0, 8 > −5, 8 > −93, 8 > −1235, etc.
2. Every negative rational number is less than 0 (zero) and is less than every positive number :
   e.g. −8 < 0, −8 < 5, −8 < 93, −8 < 123, etc.

3. Zero (0) is greater than every negative number and is smaller than every positive number :

   e.g. $0 > -5$, $0 > -\frac{3}{20}$, $0 < 5$, $0 < \frac{3}{20}$, etc.

Example 3 :

Compare $\frac{5}{7}$ and $-\frac{3}{5}$.

Solution :

$\frac{5}{7}$ is a positive rational number and $-\frac{3}{5}$ is a negative rational number.

Since, a positive rational number is always greater than 0 and greater than every negative number.

$\therefore$ **$\frac{5}{7}$ is greater than $-\frac{3}{5}$** **(Ans.)**

Example 4 :

Compare $\frac{3}{5}$ and $\frac{5}{7}$.

Solution :

First method :

1. Find the L.C.M. of denominators 5 and 7

   L.C.M. of 5 and 7 = 35

2. Make denominator of each rational number equal to L.C.M. obtained above *i.e.* equal to 35

   $$\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}$$

   and $$\frac{5}{7} = \frac{5 \times 5}{7 \times 5} = \frac{25}{35}$$

3. For the same denominator, the rational number with greater numerator is greater

   $\Rightarrow$ $\frac{25}{35}$ is greater than $\frac{21}{35}$

   $\Rightarrow$ **$\frac{5}{7}$ is greater than $\frac{3}{5}$** **(Ans.)**

Second method :

Cross-multiply $\frac{a}{b}$ and $\frac{c}{d}$; we get $a \times d$ and $b \times c$

1. If $a \times d$ is greater than $b \times c \Rightarrow \frac{a}{b}$ is greater than $\frac{c}{d}$ *i.e.* $\frac{a}{b} > \frac{c}{d}$.
2. If $a \times d$ is less than $b \times c \Rightarrow \frac{a}{b}$ is less than $\frac{c}{d}$ *i.e.* $\frac{a}{b} < \frac{c}{d}$.

For $\frac{3}{5}$ and $\frac{5}{7}$

we get : 3 × 7 and 5 × 5

$\Rightarrow$ 21 and 25

$\because$ 21 < 25

$\Rightarrow$ $\frac{3}{5}$ **is smaller than** $\frac{5}{7}$ **(Ans.)**

Example 5 :

Compare $\frac{-3}{7}$ and $\frac{8}{-15}$.

Solution :

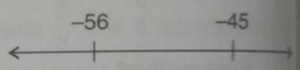
Express each given rational number with a positive denominator

$\therefore \frac{-3}{7}$ and $\frac{8}{-15} \Rightarrow \frac{-3}{7}$ and $\frac{-8}{15}$

Now, make the denominators of both the rational numbers the same. For this

$\frac{-3}{7}$ and $\frac{-8}{15} = \frac{-3\times15}{7\times15}$ and $\frac{-8\times7}{15\times7}$ [$\because$ L.C.M. of 7 and 15 = 7 × 15 = 105

$= \frac{-45}{105}$ and $\frac{-56}{105}$



Since, –45 is greater than –56

$\Rightarrow \frac{-45}{105}$ is greater than $\frac{-56}{105}$

$\Rightarrow \frac{-3}{7}$ **is greater than** $\frac{-8}{15}$ **(Ans.)**

Example 6 :

Arrange the rational numbers $-\frac{9}{10}, \frac{7}{-8}$ and $\frac{-3}{4}$ in ascending order.

Solution :

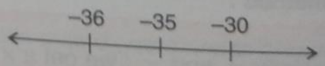
$-\frac{9}{10}, \frac{7}{-8}$ and $\frac{-3}{4} = \frac{-9}{10}, \frac{-7}{8}$ and $\frac{-3}{4}$

$= \frac{-9\times4}{10\times4}, \frac{-7\times5}{8\times5}$ and $\frac{-3\times10}{4\times10}$ [L.C.M. of 10, 8 and 4 = 40]

$= \frac{-36}{40}, \frac{-35}{40}$ and $\frac{-30}{40}$

Since, –36 < – 35 < – 30

$\therefore \frac{-36}{40} < \frac{-35}{40} < \frac{-30}{40}$



$\Rightarrow \frac{-9}{10} < \frac{7}{-8} < \frac{-3}{4}$ **(Ans.)**

Example 7 :

Arrange the rational numbers $\frac{5}{-8}, \frac{-7}{12}$ and $\frac{13}{-24}$ in descending order.

Solution :

$$\frac{5}{-8}, \frac{-7}{12} \text{ and } \frac{13}{-24} = \frac{-5}{8}, \frac{-7}{12} \text{ and } \frac{-13}{24}$$

$$= \frac{-5\times3}{8\times3}, \frac{-7\times2}{12\times2} \text{ and } \frac{-13}{24} \quad \text{[L.C.M. of 8, 12 and 24 is 24]}$$

i.e., $\frac{-15}{24}, \frac{-14}{24}$ and $\frac{-13}{24}$ in which $-13 > -14 > -15$

$\Rightarrow \frac{-13}{24} > \frac{-14}{24} > \frac{-15}{24}$

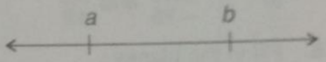
$\Rightarrow \frac{13}{-24} > -\frac{7}{12} > \frac{5}{-8}$ **(Ans.)**

Important :

For any two rational numbers *a* and *b*, marked on a number line, if :

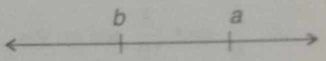
1. *a* is on the left of *b*

   then *a* is smaller than *b* *i.e.* **a < b**



2. *a* is on the right of *b*

   then *a* is greater than *b* *i.e.* **a > b**

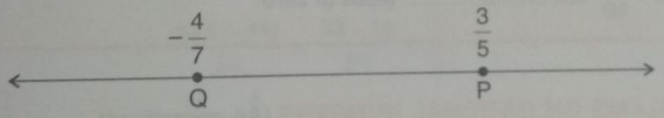


Example 8 :

Using number line, compare the numbers $\frac{3}{5}$ and $-\frac{4}{7}$.

Solution :

Mark given rational numbers roughly on a number line as shown below :



Since, P is on the right of Q.

$\Rightarrow$ **$\frac{3}{5}$ is greater than $-\frac{4}{7}$** **(Ans.)**

EXERCISE 2(B)

1. Mark the following pairs of rational numbers on the separate number lines :

(i) $\frac{3}{4}$ and $-\frac{1}{4}$ (ii) $\frac{2}{5}$ and $\frac{-3}{5}$ (iii) $\frac{5}{6}$ and $-\frac{2}{3}$

(iv) $\frac{2}{5}$ and $-\frac{4}{5}$ (v) $\frac{1}{4}$ and $-\frac{5}{4}$

2. Compare :

(i) $\frac{3}{5}$ and $\frac{5}{7}$ (ii) $\frac{-7}{2}$ and $\frac{5}{2}$ (iii) -3 and $2\frac{3}{4}$

(iv) $-1\frac{1}{2}$ and 0 (v) 0 and $\frac{3}{4}$ (vi) 3 and -1

3. Compare :

(i) $-\frac{1}{4}$ and 0 (ii) $\frac{1}{4}$ and 0 (iii) $-\frac{3}{8}$ and $\frac{2}{5}$

(iv) $\frac{-5}{8}$ and $\frac{7}{-12}$ (v) $\frac{5}{-9}$ and $\frac{-5}{-9}$ (vi) $\frac{-7}{8}$ and $\frac{5}{-6}$

(vii) $\frac{2}{7}$ and $\frac{-3}{-8}$

4. Arrange the given rational numbers in ascending order :

(i) $\frac{7}{10}, \frac{-11}{-30}$ and $\frac{5}{-15}$ (ii) $\frac{4}{-9}, \frac{-5}{12}$ and $\frac{2}{-3}$

5. Arrange the given rational numbers in descending order :

(i) $\frac{5}{8}, \frac{13}{-16}$ and $\frac{-7}{12}$ (ii) $\frac{3}{-10}, \frac{-13}{30}$ and $\frac{8}{-20}$

6. Fill in the blanks :

(i) $\frac{5}{8}$ and $\frac{3}{10}$ are on the .... side of zero.

(ii) $-\frac{5}{8}$ and $\frac{3}{10}$ are on the .... sides of zero.

(iii) $-\frac{5}{8}$ and $-\frac{3}{10}$ are on the .... side of zero.

(iv) $\frac{5}{8}$ and $-\frac{3}{10}$ are on the .... sides of zero.

2.5 PROBLEMS ON RATIONAL NUMBERS (All operations) :

1. Addition of Rational Numbers :

Case 1 : When denominators are equal :

- Keeping the denominator same, add the numerators.
- If required, express the rational number obtained in its lowest terms.

Thus :

(a) $\frac{4}{15} + \frac{8}{15} = \frac{4+8}{15} = \frac{12}{15} = \frac{4}{5}$

(b) $\frac{-4}{15} + \frac{8}{15} = \frac{(-4)+8}{15} = \frac{4}{15}$

(c) $\frac{-4}{15} + \frac{8}{-15} = \frac{-4}{15} + \frac{-8}{15} = \frac{(-4)+(-8)}{15} = \frac{-12}{15} = -\frac{4}{5}$

(d) $\frac{4}{15} + \frac{8}{-15} = \frac{4}{15} + \frac{-8}{15} = \frac{4+(-8)}{15} = \frac{-4}{15}$

Case 2 : When denominators are unequal :

Make the denominators of all the given rational numbers the same and then proceed as case 1, given above. Thus :

(a) $$\frac{-2}{3}+\frac{3}{4} = \frac{-2\times4}{3\times4}+\frac{3\times3}{4\times3}$$ [∵ L.C.M. of 3 and 4 is 12
So, make each denominator equal to 12]

$$= \frac{-8}{12}+\frac{9}{12}$$

$$= \frac{(-8)+9}{12} = \frac{-8+9}{12} = \frac{1}{12}$$

(b) $$\frac{-3}{5}+\frac{7}{-10} = \frac{-3}{5}+\frac{-7}{10}$$

$$= \frac{-3\times2}{5\times2}+\frac{-7}{10}$$ [∵ L.C.M. of 5 and 10 is 10]

$$= \frac{-6}{10}+\frac{-7}{10}$$

$$= \frac{(-6)+(-7)}{10} = \frac{-13}{10}$$

(c) $$\frac{11}{18}+\frac{7}{-27} = \frac{11}{18}+\frac{-7}{27}$$

$$= \frac{11\times3}{18\times3}+\frac{-7\times2}{27\times2}$$ [∵ L.C.M. of 18 and 27 is 54]

$$= \frac{33}{54}+\frac{-14}{54}$$

$$= \frac{33+(-14)}{54} = \frac{33-14}{54} = \frac{19}{54}$$

(d) $$\frac{9}{-16}+\frac{-5}{-12} = \frac{-9}{16}+\frac{5}{12}$$

$$= \frac{-9\times3}{16\times3}+\frac{5\times4}{12\times4}$$ [∵ L.C.M. of 16 and 12 is 48]

$$= \frac{-27}{48}+\frac{20}{48}$$

$$= \frac{-27+20}{48} = \frac{-7}{48}$$

(e) $$\frac{-4}{9}+\frac{7}{-12}+\frac{7}{18} = \frac{-4}{9}+\frac{-7}{12}+\frac{7}{18}$$

$$= \frac{-4\times4}{9\times4}+\frac{-7\times3}{12\times3}+\frac{7\times2}{18\times2}$$ [∵ L.C.M. of 9, 12 and 18 is 36]

$$= \frac{-16}{36}+\frac{-21}{36}+\frac{14}{36}$$

$$= \frac{-16+(-21)+14}{36} = \frac{-16-21+14}{36} = \frac{-37+14}{36} = \frac{-23}{36}$$

(f) $\frac{5}{-27} + \frac{8}{9} + \frac{-7}{18} = \frac{-5}{27} + \frac{8}{9} + \frac{-7}{18}$

$= \frac{-5 \times 2}{27 \times 2} + \frac{8 \times 6}{9 \times 6} + \frac{-7 \times 3}{18 \times 3}$ [∵ L.C.M. of 27, 9 and 18 is 54

$= \frac{-10}{54} + \frac{48}{54} + \frac{-21}{54}$

$= \frac{-10+48-21}{54} = \frac{-31+48}{54} = \frac{17}{54}$

2. Subtraction of Rational Numbers :

Case 1 : When denominators are equal :

(a) $\frac{5}{7} - \frac{4}{7} = \frac{5-4}{7} = \frac{1}{7}$

(b) $\frac{9}{13} - \frac{6}{13} = \frac{9-6}{13} = \frac{3}{13}$

(c) $\frac{12}{25} - \frac{8}{25} = \frac{12-8}{25} = \frac{4}{25}$

In general, for any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$.

$\frac{a}{b} - \frac{c}{d} = \frac{a}{b}$ + (additive inverse of $\frac{c}{d}$)

$= \frac{a}{b} + \frac{-c}{d}$

Additive inverse of $\frac{8}{9}$ is $\frac{-8}{9}$, additive inverse of $\frac{-4}{7}$ is $\frac{4}{7}$ and so on.

(a) $\frac{3}{4} - \frac{2}{5} = \frac{3}{4}$ + (additive inverse of $\frac{2}{5}$)

$= \frac{3}{4} + \frac{-2}{5}$

$= \frac{3 \times 5}{4 \times 5} + \frac{-2 \times 4}{5 \times 4}$ [∵ L.C.M. of 4 and 5 is 20

$= \quad + \frac{-8}{20} = \frac{15+(-8)}{20} = \frac{15-8}{20} = \frac{7}{20}$

(b) $-\frac{2}{3} - \left(-\frac{3}{5}\right) = -\frac{2}{3}$ + (additive inverse of $-\frac{3}{5}$)

$= -\frac{2}{3} + \frac{3}{5}$

$= \frac{-2 \times 5}{3 \times 5} + \frac{3 \times 3}{5 \times 3}$ [∵ L.C.M. of 3 and 5 is 15

$= \frac{-10}{15} + \frac{9}{15} = \frac{-10+9}{15} = \frac{-1}{15}$

(c) $\frac{-2}{1} - \frac{7}{12} = \frac{-2}{1} + \left(\text{additive inverse of } \frac{7}{12}\right)$

$= \frac{-2}{1} + \left(\frac{-7}{12}\right)$

$= \frac{-2\times12}{1\times12} + \frac{-7}{12}$ [∵ L.C.M. of 1and 12 is 12]

$= \frac{-24}{12} + \frac{-7}{12} = \frac{-24-7}{12} = \frac{-31}{12}$

Direct method :

(a) $-3 - \frac{4}{7} = \frac{-3}{1} - \frac{4}{7}$

$= \frac{-3\times7}{1\times7} - \frac{4}{7}$

$= \frac{-21}{7} - \frac{4}{7} = \frac{-21-4}{7} = \frac{-25}{7}$

(b) $-\frac{5}{6} - \frac{5}{-3} = -\frac{5}{6} + \frac{5}{3}$

$= -\frac{5}{6} + \frac{5\times2}{3\times2} = \frac{-5}{6} + \frac{10}{6} = \frac{-5+10}{6} = \frac{5}{6}$

(c) $-\frac{5}{8} - (-3) = -\frac{5}{8} + 3 = -\frac{5}{8} + \frac{3\times8}{8}$

$= -\frac{5}{8} + \frac{24}{8} = \frac{-5+24}{8} = \frac{19}{8}$

Example 9 :

The sum of two rational numbers is –4. If one of them is $\frac{-13}{25}$, find the other.

Solution :

∵ Sum of two rational numbers = –4

and, one of them = $\frac{-13}{25}$

∴ **The other rational number** $= -4 - \left(\frac{-13}{25}\right)$

$= -4 + \frac{13}{25}$

$= \frac{-4\times25}{25} + \frac{13}{25}$

$= \frac{-100}{25} + \frac{13}{25} = \frac{-100+13}{25} = \frac{-87}{25}$ **(Ans.)**

Example 10 :

What should be added to $\frac{-5}{9}$ to get $\frac{-5}{6}$?

Solution :

Let the required rational number be x

$$\therefore \quad \frac{-5}{9} + x = \frac{-5}{6} \Rightarrow x = \left(\frac{-5}{6}\right) - \left(\frac{-5}{9}\right)$$

$$= -\frac{5}{6} + \frac{5}{9}$$

$$= \frac{-5\times3}{6\times3} + \frac{5\times2}{9\times2} \qquad [\because \text{L.C.M. of 6 and 9 is 1}$$

$$= \frac{-15}{18} + \frac{10}{18}$$

$$= \frac{-15+10}{18} = \frac{-5}{18} \qquad \text{(Ans}$$

Example 11 :

What sould be subtracted from $-\frac{5}{6}$ to get 1?

Solution :

The required rational number

$$= -\frac{5}{6} - 1$$

$$= -\frac{5}{6} - \frac{6}{6}$$

$$= \frac{-5-6}{6} = \frac{-11}{6} \qquad \text{(Ans.)}$$

Let x be subtracted

$$\therefore \quad -\frac{5}{6} - x = 1$$

$$\Rightarrow \quad -\frac{5}{6} - 1 = x$$

$$i.e. \quad x = -\frac{5}{6} - 1$$

$$= -\frac{5}{6} - \frac{6}{6} = \frac{-5-6}{6} =$$

EXERCISE 2(C)

1. Add :

(i) $\frac{7}{5}$ and $\frac{2}{5}$

(ii) $\frac{-4}{9}$ and $\frac{2}{9}$

(iii) $\frac{5}{-12}$ and $\frac{1}{12}$

(iv) $\frac{4}{-15}$ and $\frac{-7}{-15}$

(v) $\frac{-7}{25}$ and $\frac{9}{-25}$

(vi) $\frac{-7}{26}$ and $\frac{7}{-26}$

2. Add :

(i) $\frac{-2}{5}$ and $\frac{3}{7}$

(ii) $\frac{-5}{6}$ and $\frac{4}{9}$

(iii) -3 and $\frac{2}{3}$

(iv) $\frac{-5}{9}$ and $\frac{7}{18}$

(v) $\frac{-7}{24}$ and $\frac{-5}{48}$

(vi) $\frac{1}{-18}$ and $\frac{5}{-27}$

(vii) $\frac{-9}{25}$ and $\frac{1}{-75}$

(viii) $\frac{13}{-16}$ and $\frac{-11}{24}$

(ix) $\frac{-9}{-16}$ and $\frac{-11}{8}$

3. Evaluate :

(i) $\frac{-2}{5}+\frac{3}{5}+\frac{-1}{5}$ (ii) $\frac{-8}{9}+\frac{4}{9}+\frac{-2}{9}$ (iii) $\frac{5}{-24}+\frac{-1}{8}+\frac{3}{16}$

(iv) $\frac{-7}{6}+\frac{4}{-15}+\frac{-4}{-30}$ (v) $-2+\frac{2}{5}+\frac{-2}{15}$ (vi) $\frac{-11}{12}+\frac{5}{16}+\frac{-3}{8}$

4. Evaluate :

(i) $-\frac{11}{18}+\frac{-3}{9}+\frac{2}{-3}$ (ii) $\frac{-9}{4}+\frac{13}{3}+\frac{25}{6}$ (iii) $-5+\frac{5}{-8}+\frac{-5}{-12}$

(iv) $-\frac{2}{3}+\frac{5}{2}+2$ (v) $5+\frac{-3}{4}+\frac{-5}{8}$

5. Subtract :

(i) $\frac{2}{9}$ from $\frac{5}{9}$ (ii) $\frac{-6}{11}$ from $\frac{-3}{-11}$ (iii) $\frac{-2}{15}$ from $\frac{-8}{15}$

(iv) $\frac{11}{18}$ from $\frac{-5}{18}$ (v) $\frac{-4}{11}$ from -2.

6. Subtract :

(i) $-\frac{3}{10}$ from $\frac{1}{5}$ (ii) $\frac{-6}{25}$ from $\frac{-8}{5}$ (iii) $\frac{-7}{4}$ from -2

(iv) $\frac{-16}{21}$ from 1 (v) $\frac{-8}{15}$ from 0 (vi) 0 from $\frac{-3}{8}$

(vii) -2 from $\frac{-3}{10}$ (viii) $\frac{5}{8}$ from $\frac{-5}{16}$ (ix) 4 from $-\frac{3}{13}$

7. The sum of two rational numbers is $\frac{11}{24}$. If one of them is $\frac{3}{8}$, find the other.

8. The sum of two rational numbers is $\frac{-7}{12}$. If one of them is $\frac{13}{24}$, find the other.

9. The sum of two rational numbers is -4. If one of them is $-\frac{13}{12}$, find the other.

10. What should be added to $-\frac{3}{16}$ to get $\frac{11}{24}$?

11. What should be added to $\frac{-3}{5}$ to get 2 ?

12. What should be subtracted from $\frac{-4}{5}$ to get 1 ?

13. The sum of two numbers is $-\frac{6}{5}$. If one of them is -2, find the other.

14. What should be added to $\frac{-7}{12}$ to get $\frac{3}{8}$?

15. What should be subtracted from $\frac{5}{9}$ to get $\frac{9}{5}$?

3. Multiplication of Rational Numbers

Product (multiplication) of two or more rational numbers

$$= \frac{\text{Product of their numerators}}{\text{Product of their denominators}}$$

For example :

(a) $\frac{5}{6}\times\left(\frac{-3}{4}\right)=\frac{5\times(-3)}{6\times4}=\frac{-15}{24}=\frac{-5}{8}$

(b) $\left(\frac{-3}{8}\right)\times3=\frac{-3}{8}\times\frac{3}{1}=\frac{-3\times3}{8\times1}=\frac{-9}{8}$

(c) $\left(\frac{-36}{7}\right)\times\left(\frac{28}{-9}\right)=\frac{-36\times28}{7\times-9}=\frac{-36\times28}{-7\times9}=\frac{\overset{4}{\cancel{36}}\times\overset{4}{\cancel{28}}}{\cancel{7}\times\cancel{9}}=4\times4=16$

(d) $\frac{-8}{7}\times\frac{14}{5}=\frac{-8\times\cancel{14}^{2}}{\cancel{7}\times5}=\frac{-8\times2}{5}=\frac{-16}{5}$

(e) $\frac{4}{9}\times-3=\frac{4}{9}\times\frac{-3}{1}=\frac{4\times(-3)}{9\times1}=\frac{-\cancel{12}^{4}}{\cancel{9}_{3}}=-\frac{4}{3}$

(f) $-24\times\frac{-5}{8}=\frac{-24}{1}\times\frac{-5}{8}=\frac{-24\times-5}{8}=\frac{120}{8}=15$

(g) $\frac{-4}{9}\times\frac{13}{-20}=\frac{-4}{9}\times\frac{-13}{20}=\frac{-4\times-13}{9\times20}=\frac{\overset{13}{\cancel{52}}}{\cancel{180}_{45}}=\frac{13}{45}$

(h) $\left(\frac{-3}{2}\times\frac{4}{5}\right)+\left(\frac{9}{5}\times\frac{-10}{3}\right)-\left(\frac{1}{2}\times\frac{3}{4}\right)$

$$=\frac{-12}{10}+\frac{-90}{15}-\frac{3}{8}$$

$$=-\frac{6}{5}+(-6)-\frac{3}{8}$$

$$=-\frac{6}{5}-\frac{6}{1}-\frac{3}{8}$$

$$=-\frac{6\times8}{5\times8}-\frac{6\times40}{40}-\frac{3\times5}{8\times5}$$ [L.C.M. of 5, 1 and 8 is 40]

$$=-\frac{48}{40}-\frac{240}{40}-\frac{15}{40}$$

$$=\frac{-48-240-15}{40}=\frac{-303}{40}$$

Example 12 :

A cyclist moves with a speed of $7\frac{2}{5}$ km per hour. How much distance will he cover in $2\frac{1}{3}$ hours?

Solution :

$$\because \quad \text{Speed} = 7\frac{2}{5} \text{ km per hour}$$

$$= \frac{37}{5} \text{ km per hour}$$

And, $\quad$ time taken $= 2\frac{1}{3}$ hours $= \frac{7}{3}$ hours

$\therefore$ **Distance covered** $=$ Speed × time

$$= \frac{37}{5} \times \frac{7}{3} \text{ km}$$

$$= \frac{259}{15} \text{ km} = 17\frac{4}{15} \text{ km}$$ **(Ans.)**

4. Multiplicative Inverse (Reciprocal)

(a) Multiplicative inverse of $\frac{3}{4}$ is $\frac{4}{3} \Rightarrow$ Reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

(b) Multiplicative inverse of $\frac{-4}{5}$ is $\frac{5}{-4} \Rightarrow$ Reciprocal of $\frac{-4}{5}$ is $\frac{5}{-4}$.

(c) Multiplicative inverse (reciprocal) of $\frac{3}{-8}$ is $\frac{-8}{3}$.

(d) Multiplicative inverse (reciprocal) of $\frac{-6}{13}$ is $\frac{13}{-6} = \frac{-13}{6}$.

5. Division of Rational Numbers

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d} \neq 0$, then,

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times (\text{multiplicative inverse of } \frac{c}{d})$$

$$= \frac{a}{b} \times \frac{d}{c}$$

For example :

(a) $\frac{4}{25} \div \frac{3}{5} = \frac{4}{25} \times \frac{5}{3}$ $\quad$ [Reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$]

$= \frac{4}{5\times3} = \frac{4}{15}$ **(Ans.)**

(b) $\frac{2}{7} \div \frac{-8}{35} = \frac{2}{7} \times \frac{35}{-8} = \frac{2\times5}{-8} = -\frac{10}{8} = -\frac{5}{4}$ **(Ans.)**

(c) $-\frac{4}{3} \div \frac{16}{21} = -\frac{4}{3} \times \frac{21}{16} = -\frac{1\times7}{1\times4} = -\frac{7}{4}$ **(Ans.)**

(d) $-\frac{4}{3} \div \frac{-16}{21} = -\frac{4}{3} \times \frac{21}{-16} = \frac{1\times7}{1\times4} = \frac{7}{4}$ **(Ans.)**

(e) $-\frac{3}{10} \div \frac{-9}{25} = -\frac{3}{10} \times \frac{25}{-9} = \frac{1\times5}{2\times3} = \frac{5}{6}$ **(Ans.)**

Example 14 :

The product of two rational numbers is $\frac{12}{5}$. If one of the numbers is $-\frac{6}{7}$, find the other.

Solution :

$\because$ Product of two rational numbers $= \frac{12}{5}$

and, one of these two numbers $= -\frac{6}{7}$

$\therefore$ The other number $= \frac{12}{5} \div \left(-\frac{6}{7}\right)$

$$= \frac{12}{5} \times \left(-\frac{7}{6}\right) = -\frac{12}{5} \times \frac{7}{6} = -\frac{2 \times 7}{5 \times 1} = -\frac{14}{5}$$ (Ans.)

Example 15 :

Evaluate : $\left(3\frac{2}{5} + 2\frac{1}{10}\right) \div \left(4\frac{3}{5} - 3\frac{3}{10}\right)$.

Solution :

$$\left(3\frac{2}{5} + 2\frac{1}{10}\right) \div \left(4\frac{3}{5} - 3\frac{3}{10}\right) = \left(\frac{17}{5} + \frac{21}{10}\right) \div \left(\frac{23}{5} - \frac{33}{10}\right)$$

$$= \left(\frac{17 \times 2 + 21}{10}\right) \div \left(\frac{23 \times 2 - 33}{10}\right)$$

$$= \frac{34 + 21}{10} \div \frac{46 - 33}{10}$$

$$= \frac{55}{10} \div \frac{13}{10} = \frac{55}{10} \times \frac{10}{13} = \frac{55}{13} = 4\frac{3}{13}$$ (Ans.)

EXERCISE 2(D)

1. Evaluate :

(i) $\frac{5}{4} \times \frac{3}{7}$ (ii) $\frac{2}{3} \times -\frac{6}{7}$ (iii) $\left(\frac{-12}{5}\right) \times \left(\frac{10}{-3}\right)$

(iv) $-\frac{45}{39} \times \frac{-13}{15}$ (v) $3\frac{1}{8} \times \left(-2\frac{2}{5}\right)$ (vi) $2\frac{14}{25} \times \left(\frac{-5}{16}\right)$

(vii) $\left(\frac{-8}{9}\right) \times \left(\frac{-3}{16}\right)$ (viii) $\left(\frac{5}{-27}\right) \times \left(\frac{-9}{20}\right)$

2. Multiply :

(i) $\frac{3}{25}$ and $\frac{4}{5}$ (ii) $1\frac{1}{8}$ and $10\frac{2}{3}$ (iii) $6\frac{2}{3}$ and $\frac{-3}{8}$

(iv) $\frac{-13}{15}$ and $\frac{-25}{26}$ (v) $1\frac{1}{6}$ and 18 (vi) $2\frac{1}{14}$ and -7

(vii) $5\frac{1}{8}$ and -16 (viii) 35 and $\frac{-18}{25}$ (ix) $6\frac{2}{3}$ and $-\frac{3}{8}$

(x) $3\frac{3}{5}$ and -10 (xi) $\frac{27}{28}$ and -14 (xii) -24 and $\frac{5}{16}$

3. Evaluate :

(i) $\left(-6\times\frac{5}{18}\right)-\left(-4\frac{2}{9}\right)$

(ii) $\left(\frac{7}{8}\times\frac{8}{7}\right)+\left(\frac{-5}{9}\right)\times\left(\frac{6}{-25}\right)$

(iii) $\left(\frac{11}{-9}\times\frac{21}{44}\right)+\left(\frac{-5}{9}\right)\times\left(\frac{63}{-100}\right)$

(iv) $\left(\frac{-5}{9}\times\frac{6}{-25}\right)+\left(\frac{24}{21}\times\frac{7}{8}\right)$

(v) $\left(\frac{-35}{39}\times\frac{-13}{7}\right)-\left(\frac{7}{90}\times\frac{-18}{14}\right)$

(vi) $\left(\frac{-4}{5}\times\frac{3}{2}\right)+\left(\frac{9}{-5}\times\frac{10}{3}\right)-\left(\frac{-3}{2}\times\frac{-1}{4}\right)$

4. Find the cost of $3\frac{1}{2}$ m cloth, if one metre cloth costs ₹ $325\frac{1}{2}$.

5. A bus is moving with a speed of $65\frac{1}{2}$ km per hour. How much distance will it cover in $1\frac{1}{3}$ hours.

6. Divide :

(i) $\frac{15}{28}$ by $\frac{3}{4}$

(ii) $\frac{-20}{9}$ by $\frac{-5}{9}$

(iii) $\frac{16}{-5}$ by $\frac{-8}{7}$

(iv) -7 by $\frac{-14}{5}$

(v) -14 by $\frac{7}{-2}$

(vi) $\frac{-22}{9}$ by $\frac{11}{18}$

(vii) 35 by $\frac{-7}{9}$

(viii) $\frac{21}{44}$ by $-\frac{11}{9}$

7. Evaluate :

(i) $3\frac{5}{12}+1\frac{2}{3}$

(ii) $3\frac{5}{12}-1\frac{2}{3}$

(iii) $\left(3\frac{5}{12}+1\frac{2}{3}\right)\div\left(3\frac{5}{12}-1\frac{2}{3}\right)$

8. The product of two numbers is 14. If one of the numbers is $\frac{-8}{7}$, find the other.

9. The cost of 11 pens is ₹ $24\frac{3}{4}$. Find the cost of one pen.

10. If 6 identical articles can be bought for ₹ $2\frac{6}{17}$. Find the cost of each article.

11. By what number should $\frac{-3}{8}$ be multiplied so that the product is $\frac{-9}{16}$?

12. By what number should $\frac{-5}{7}$ be divided so that the result is $\frac{-15}{28}$?

13. Evaluate : $\left(\frac{32}{15}+\frac{8}{5}\right)\div\left(\frac{32}{15}-\frac{8}{5}\right)$.

14. Seven equal pieces are made out of a rope of $21\frac{5}{7}$ m. Find the length of each piece.

EXERCISE 2(E)

1. Evaluate :

(i) $\frac{-2}{3} + \frac{3}{4}$ (ii) $\frac{7}{-27} + \frac{11}{18}$ (iii) $\frac{-3}{8} + \frac{-5}{12}$

(iv) $\frac{9}{-16} + \frac{-5}{-12}$ (v) $\frac{-5}{9} + \frac{-7}{12} + \frac{11}{18}$ (vi) $\frac{7}{-26} + \frac{16}{39}$

(vii) $-\frac{2}{3} - \left(\frac{-5}{7}\right)$ (viii) $-\frac{5}{7} - \left(-\frac{3}{8}\right)$ (ix) $\frac{7}{26} + 2 + \frac{-11}{13}$

(x) $-1 + \frac{2}{-3} + \frac{5}{6}$

2. The sum of two rational numbers is $\frac{-3}{8}$. If one of them is $\frac{3}{16}$, find the other.

3. The sum of two rational numbers is –5. If one of them is $\frac{-52}{25}$, find the other.

4. What rational number should be added to $-\frac{3}{16}$ to get $\frac{11}{24}$?

5. What rational number should be added to $-\frac{3}{5}$ to get 2 ?

6. What rational number should be subtracted from $-\frac{5}{12}$ to get $\frac{5}{24}$?

7. What rational number should be subtracted from $\frac{5}{8}$ to get $\frac{8}{5}$?

8. Evaluate :

(i) $\left(\frac{7}{8} \times \frac{24}{21}\right) + \left(\frac{-5}{9} \times \frac{6}{-25}\right)$ (ii) $\left(\frac{8}{15} \times \frac{-25}{16}\right) + \left(\frac{-18}{35} \times \frac{5}{6}\right)$

(iii) $\left(\frac{18}{33} \times \frac{-22}{27}\right) - \left(\frac{13}{25} \times \frac{-75}{26}\right)$ (iv) $\left(\frac{-13}{7} \times \frac{-35}{39}\right) - \left(\frac{-7}{45} \times \frac{9}{14}\right)$

9. The product of two rational numbers is 24. If one of them is $\frac{-36}{11}$, find the other.

10. By what rational number should we multiply $\frac{20}{-9}$, so that the product may be $\frac{-5}{9}$?

FRACTION 3

(Including Problems)

3.1 BASIC CONCEPT

If an apple is divided into five equal parts; each part is said to be one-fifth $\left(\frac{1}{5}\right)$ of the whole apple. And, if out of these five equal parts, 2 parts are eaten; we say two-fifths $\left(\frac{2}{5}\right)$ of the apple is eaten or three-fifths $\left(\frac{3}{5}\right)$ of the apple is left.

The numbers $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{3}{5}$ used in the statement, given above, are called **fractions**. *Each* of these fractions *indicates a part of the whole.*

In fraction $\frac{a}{b}$, ***a*** is called the **numerator** and ***b*** is called the **denominator** of the fraction.

$\therefore$ FRACTION = $\frac{\text{Numerator}}{\text{Denominator}}$

Every fraction can be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$ *i.e.* denominator is not equal to zero.

3.2 CLASSIFICATION OF FRACTIONS

Types of fractions	Condition	Examples
1. ***Decimal fraction***	denominator is 10 or higher power of 10.	$\frac{1}{10}, \frac{3}{100}, \frac{15}{1000}, \frac{8}{10^5}, \ldots\ldots$
2. ***Vulgar fraction***	denominator is other than 10, 100, 1000, etc.	$\frac{2}{5}, \frac{4}{7}, \frac{8}{19}, \frac{23}{107}, \ldots\ldots$
3. ***Proper fraction***	denominator is greater than its numerator.	$\frac{4}{5}, \frac{3}{7}, \frac{101}{235}, \ldots\ldots$
4. ***Improper fraction***	denominator is less than its numerator.	$\frac{7}{5}, \frac{18}{13}, \frac{181}{60}, \ldots\ldots$
5. ***Mixed fraction***	consists of an integer and a proper fraction.	$2\frac{5}{7}, 1\frac{3}{5}, 10\frac{1}{9}, \ldots\ldots$

If the numerator is equal to the denominator, the fraction is equal to **unity** (one).

e.g. $\frac{4}{4} = 1$, $\frac{-3}{-3} = 1$, $\frac{49}{49} = 1$ and so on.

Important : (a) $\frac{\mathbf{7}}{\mathbf{20}} = \frac{7 \times 5}{20 \times 5} = \frac{35}{100}$, **a decimal fraction.**

(b) $\frac{\mathbf{81}}{\mathbf{500}} = \frac{81 \times 2}{500 \times 2} = \frac{162}{1000}$, **a decimal fraction.**

$\therefore$ If the denominator of a fraction can be expressed as 10 or as some higher power of 10, it is a decimal fraction.

Example 1 :

(a) Convert : (i) $3\frac{2}{7}$ (ii) $2\frac{5}{8}$ into improper fractions.

(b) Convert : (i) $\frac{11}{4}$ (ii) $\frac{19}{5}$ into mixed fractions.

Solution :

(a) (i) $3\frac{2}{7} = \frac{3\times7+2}{7} = \frac{23}{7}$ (Ans.)

$$\boxed{\text{Given mixed fraction} = \frac{\text{Integral part} \times \text{Denominator} + \text{Numerator}}{\text{Denominator}}}$$

(ii) $2\frac{5}{8} = \frac{2\times8+5}{8} = \frac{16+5}{8} = \frac{21}{8}$ (Ans.)

(b) (i) $\frac{11}{4} = \frac{2\times4+3}{4}$ $\quad\because\quad \begin{array}{r}4\,\overline{)\,11\,}\,(2\\ \underline{8}\\ 3\end{array}$

$= 2 + \frac{3}{4} = 2\frac{3}{4}$ (Ans.)

(ii) $\frac{19}{5} = \frac{3\times5+4}{5}$ $\quad\because\quad \begin{array}{r}5\,\overline{)\,19\,}\,(3\\ \underline{15}\\ 4\end{array}$

$= 3 + \frac{4}{5} = 3\frac{4}{5}$ (Ans.)

1. The value of a fraction remains the same if both its numerator and denominator are (i) multiplied or (ii) divided by the same non-zero number.

   e.g. (i) $\frac{5}{8} = \frac{5\times2}{8\times2} = \frac{10}{16}$; $\quad \frac{3}{7} = \frac{3\times5}{7\times5} = \frac{15}{35}$ and so on.

   (ii) $\frac{10}{16} = \frac{10\div2}{16\div2} = \frac{5}{8}$; $\quad \frac{15}{35} = \frac{15\div5}{35\div5} = \frac{3}{7}$ and so on.

2. *A fraction must always be expressed in its lowest term.*

3.3 REDUCING A GIVEN FRACTION TO ITS LOWEST TERM

Steps : First of all find H.C.F. of both the terms (numerator and denominator) of the given fraction. Then divide both terms by their H.C.F.

Example 2 :

Reduce : (i) $\frac{48}{60}$ (ii) $\frac{18}{27}$ to their lowest terms.

Solution :

(i) Since, H.C.F. of terms 48 and 60 = 12.

$\therefore \frac{48}{60} = \frac{48\div12}{60\div12}$ [Dividing each term by 12]

$= \frac{4}{5}$ (Ans.)

(ii) Since, H.C.F. of 18 and 27 is 9 $\therefore \frac{18}{27} = \frac{18 \div 9}{27 \div 9} = \frac{2}{3}$ **(Ans.)**

Alternative Method :

Resolve both the numerator and the denominator into prime factors, then cancel out the common factors among both.

Since, $48 = 2 \times 2 \times 2 \times 2 \times 3$ and $60 = 2 \times 2 \times 3 \times 5$

$$\therefore \quad \frac{48}{60} = \frac{\cancel{2} \times \cancel{2} \times 2 \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{2} \times \cancel{3} \times 5} \quad \text{[Cancelling out the common factors]}$$

$$= \frac{2 \times 2}{5} = \frac{4}{5}$$

(Ans.)

3.4 EQUIVALENT (EQUAL) FRACTIONS

Fractions having the same value are called equivalent fractions.

e.g., Since, $\frac{20}{25} = \frac{20 \div 5}{25 \div 5} = \frac{4}{5}$ and $\frac{28}{35} = \frac{28 \div 7}{35 \div 7} = \frac{4}{5}$

$\therefore$ Fractions $\frac{20}{25}$ and $\frac{28}{35}$ are equivalent, *i.e.*, $\frac{20}{25} = \frac{28}{35} = \frac{4}{5}$.

3.5 SIMPLE AND COMPLEX FRACTIONS

A fraction, whose *numerator and denominator both are integers*, is called a *simple fraction;* whereas a fraction, whose *numerator or denominator or both are not integers*, is called a *complex fraction.*

e.g. (i) Each of $\frac{3}{8}, \frac{-10}{17}, \frac{8}{-15}$, etc., is a simple fraction.

(ii) Each of $\frac{5}{2/3}, \frac{1 \cdot 4}{8}, \frac{9/14}{2\frac{3}{7}}$, etc., is a complex fraction.

EXERCISE 3(A)

1. Classify each fraction given below as decimal or vulgar fraction, proper or improper fraction and mixed fraction :

(i) $\frac{3}{5}$ (ii) $\frac{11}{10}$ (iii) $\frac{13}{20}$ (iv) $\frac{18}{7}$ (v) $3\frac{2}{9}$

2. Express the following improper fractions as mixed fractions :

(i) $\frac{18}{5}$ (ii) $\frac{7}{4}$ (iii) $\frac{25}{6}$ (iv) $\frac{38}{5}$ (v) $\frac{22}{5}$

3. Express the following mixed fractions as improper fractions :

(i) $2\frac{4}{9}$ (ii) $7\frac{5}{13}$ (iii) $3\frac{1}{4}$ (iv) $2\frac{5}{48}$ (v) $12\frac{7}{11}$

4. Reduce the given fractions to lowest terms :

(i) $\frac{8}{18}$ (ii) $\frac{27}{36}$ (iii) $\frac{18}{42}$ (iv) $\frac{35}{75}$ (v) $\frac{18}{45}$

5. State true or false :

(i) $\frac{30}{40}$ and $\frac{12}{16}$ are equivalent fractions.

(ii) $\frac{10}{25}$ and $\frac{25}{10}$ are equivalent fractions.

(iii) $\frac{35}{49}, \frac{20}{28}, \frac{45}{63}$ and $\frac{100}{140}$ are equivalent fractions.

6. Distinguish each of the fractions, given below, as a *simple fraction* or a *complex fractio*

(i) $\frac{0}{8}$ (ii) $\frac{-3}{-8}$ (iii) $\frac{5}{-7}$ (iv) $\frac{3\frac{3}{5}}{18}$

(v) $\frac{-6}{2\frac{2}{5}}$ (vi) $\frac{3\frac{1}{3}}{7\frac{2}{7}}$ (vii) $\frac{-5\frac{2}{9}}{5}$ (viii) $\frac{-8}{0}$

Remember : Each of the numbers of the form $\frac{5}{0}, \frac{-7}{0}, \frac{8}{0}$, etc., is neither a simple fraction nor a complex fraction, as the division by '0' is not defined.

3.6 LIKE AND UNLIKE FRACTIONS

Fractions having the *same denominators* are called *like fractions,* whereas th *fractions with different denominators* are called *unlike fractions.*

e.g. (i) $\frac{3}{8}, \frac{5}{8}, \frac{9}{8}$, etc., are *like fractions.*

(ii) $\frac{2}{7}, \frac{5}{9}, \frac{15}{23}, \frac{24}{37}$, etc., are *unlike fractions.*

3.7 CONVERTING UNLIKE FRACTIONS INTO LIKE FRACTIONS

Steps : 1. Find the L.C.M. of the denominators of all the given fractions.

2. For each given fraction, multiply its denominator by a suitable number s that the product obtained is equal to the L.C.M. obtained in Step 1.

3. Multiply the numerator also by the same number.

Example 3 :

Change $\frac{3}{4}, \frac{3}{5}, \frac{7}{8}$ and $\frac{9}{16}$ to like fractions.

Solution :

Since, L.C.M. of the denominators 4, 5, 8 and 16 is 80.

$\therefore \quad \frac{3}{4} = \frac{3\times20}{4\times20} = \frac{60}{80}; \qquad \frac{3}{5} = \frac{3\times16}{5\times16} = \frac{48}{80}$

$\frac{7}{8} = \frac{7\times10}{8\times10} = \frac{70}{80}$; $\frac{9}{16} = \frac{9\times5}{16\times5} = \frac{45}{80}$

∴ **Required like fractions are** : $\frac{60}{80}, \frac{48}{80}, \frac{70}{80}$ and $\frac{45}{80}$ **(Ans.)**

3.8 COMPARING FRACTIONS

Steps : Convert all the given fractions *into like fractions*, then *the* ***fraction with the greater numerator is greater.***

Example 4 :

Compare the fractions : $\frac{2}{3}, \frac{3}{4}, \frac{5}{12}$ and $\frac{9}{16}$.

Solution :

∵ L.C.M. of the denominators 3, 4, 12 and 16 = 48.

$$\therefore \quad \frac{2}{3} = \frac{2\times16}{3\times16} = \frac{32}{48} \quad ; \quad \frac{3}{4} = \frac{3\times12}{4\times12} = \frac{36}{48}$$

$$\frac{5}{12} = \frac{5\times4}{12\times4} = \frac{20}{48} \quad \text{and} \quad \frac{9}{16} = \frac{9\times3}{16\times3} = \frac{27}{48}$$

Converting into like fractions

Since, the biggest numerator is 36, thus the biggest fraction is $\frac{36}{48}$ (*i.e.,* $\frac{3}{4}$).

Next one is $\frac{32}{48}$ (*i.e.,* $\frac{2}{3}$) and the smallest fraction is $\frac{20}{48}$ (*i.e.,* $\frac{5}{12}$)

∴ **Fractions in ascending order** of values are : $\frac{5}{12}, \frac{9}{16}, \frac{2}{3}$ and $\frac{3}{4}$. **(Ans.)**

i.e. $\frac{5}{12} < \frac{9}{16} < \frac{2}{3} < \frac{3}{4}$

And, **fractions in descending order** of values are : $\frac{3}{4}, \frac{2}{3}, \frac{9}{16}$ and $\frac{5}{12}$. **(Ans.)**

i.e. $\frac{3}{4} > \frac{2}{3} > \frac{9}{16} > \frac{5}{12}$

> **Ascending order** means arranging the numbers from **smallest to greatest** and **descending order** means arranging the numbers from **greatest to smallest**.

Alternate Method (By making numerators equal) **:**

Steps : 1. Convert all the given fractions into fractions of equal numerators.

2. The fraction which has a smaller denominator is greater.

Example 5 :

Compare : $\frac{2}{3}, \frac{3}{4}, \frac{5}{12}$ and $\frac{9}{16}$ by making their numerators equal.

Solution :

Step 1 : Since, L.C.M. of numerators 2, 3, 5 and 9 is 90

$$\therefore \quad \frac{2}{3} = \frac{2\times45}{3\times45} = \frac{90}{135} \quad ; \quad \frac{3}{4} = \frac{3\times30}{4\times30} = \frac{90}{120}$$

$$\frac{5}{12} = \frac{5\times18}{12\times18} = \frac{90}{216} \quad \text{and} \quad \frac{9}{16} = \frac{9\times10}{16\times10} = \frac{90}{160}$$

Step 2 : Since, $\frac{90}{120}$ has the smallest denominator, the biggest fraction is $\frac{90}{120}$ (*i.e.,* $\frac{3}{4}$).

As, $\frac{90}{216}$ has the biggest denominator, the smallest fraction is $\frac{90}{216}$ (*i.e.,* $\frac{5}{12}$).

$\therefore$ **Fractions in ascending order** are : $\frac{5}{12}, \frac{9}{16}, \frac{2}{3}$ and $\frac{3}{4}$. (Ans.)

And, **in descending order** they are : $\frac{3}{4}, \frac{2}{3}, \frac{9}{16}$ and $\frac{5}{12}$. (Ans.)

> *In order to compare two fractions, say :* $\frac{a}{b}$ *and* $\frac{c}{d}$*, find their cross-product, i.e., find* $a \times d$ *and* $b \times c$*. Then, if :*
>
> *(i)* $a \times d$ *is greater than* $b \times c \Rightarrow \frac{a}{b} > \frac{c}{d}$*,* *(ii)* $a \times d$ *is less than* $b \times c \Rightarrow \frac{a}{b} < \frac{c}{d}$*,*
>
> *(iii)* $a \times d$ *is equal to* $b \times c \Rightarrow \frac{a}{b} = \frac{c}{d}$*.*

Example 6 :

Compare the fractions : $\frac{3}{13}$ and $\frac{7}{18}$.

Solution :

Taking the cross multiplication we get : $3 \times 18 = 54$ and $7 \times 13 = 91$

Since, 3×18 (*i.e.,* 54) is *smaller* than 7×13 (*i.e.,* 91) $\therefore \frac{3}{13} < \frac{7}{18}$ (Ans.)

3.9 TO INSERT A FRACTION BETWEEN THE TWO GIVEN FRACTIONS

Steps : Add numerators of the given fractions to get the numerator of required fraction. Similarly, add their denominators to get denominator of the required fraction. Then simplify, if required.

Example 7 :

Insert one fraction between : (i) $\frac{1}{2}$ and $\frac{3}{5}$ (ii) 2 and $3\frac{1}{2}$

Solution :

(i) **A fraction between $\frac{1}{2}$ and $\frac{3}{5}$** $= \frac{1+3}{2+5}$ [Adding numerators and denominators]

$= \frac{4}{7}$ (Ans.)

> Thus, if $\frac{a}{b}$ and $\frac{c}{d}$ are two fractions then fraction $\frac{a+c}{b+d}$ lies between $\frac{a}{b}$ and $\frac{c}{d}$.
>
> **Also, 1.** If $\frac{a}{b} > \frac{c}{d}$, then $\frac{a}{b} > \frac{a+c}{b+d} > \frac{c}{d}$. **2.** If $\frac{a}{b} < \frac{c}{d}$, then $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$.

(ii) **A fraction between 2 and $3\frac{1}{2}$** = A fraction between $\frac{2}{1}$ and $\frac{7}{2}$

$= \frac{2+7}{1+2} = \frac{9}{3} = 3$ (Ans.)

Example 8 :

Insert three fractions between $\frac{1}{2}$ and $\frac{3}{5}$.

Solution :

A fraction between $\frac{1}{2}$ and $\frac{3}{5}$

$$= \frac{1+3}{2+5} = \frac{4}{7}$$

Now a fraction between $\frac{1}{2}$ and $\frac{4}{7} = \frac{1+4}{2+7} = \frac{5}{9}$

and a fraction between $\frac{4}{7}$ and $\frac{3}{5} = \frac{4+3}{7+5} = \frac{7}{12}$

$\therefore$ **Three fractions between $\frac{1}{2}$ and $\frac{3}{5} = \frac{5}{9}, \frac{4}{7}$ and $\frac{7}{12}$** **(Ans.)**

EXERCISE 3(B)

1. For each pair, given below, state whether it forms *like fractions* or *unlike fractions* :
   (i) $\frac{5}{8}$ and $\frac{7}{8}$ (ii) $\frac{8}{15}$ and $\frac{8}{21}$ (iii) $\frac{4}{9}$ and $\frac{9}{4}$
2. Convert given fractions into fractions with *equal denominators* :
   (i) $\frac{5}{6}$ and $\frac{7}{9}$ (ii) $\frac{2}{3}, \frac{5}{6}$ and $\frac{7}{12}$ (iii) $\frac{4}{5}, \frac{17}{20}, \frac{23}{40}$ and $\frac{11}{16}$
3. Convert given fractions into fractions with *equal numerators* :
   (i) $\frac{8}{9}$ and $\frac{12}{17}$ (ii) $\frac{6}{13}, \frac{15}{23}$ and $\frac{12}{17}$ (iii) $\frac{15}{19}, \frac{25}{28}, \frac{9}{11}$ and $\frac{45}{47}$
4. Put the given fractions in ascending order by making denominators equal :
   (i) $\frac{1}{3}, \frac{2}{5}, \frac{3}{4}$ and $\frac{1}{6}$ (ii) $\frac{5}{6}, \frac{7}{8}, \frac{11}{12}$ and $\frac{3}{10}$ (iii) $\frac{5}{7}, \frac{3}{8}, \frac{9}{14}$ and $\frac{20}{21}$
5. Arrange the given fractions in descending order by making numerators equal :
   (i) $\frac{5}{6}, \frac{4}{15}, \frac{8}{9}$ and $\frac{1}{3}$ (ii) $\frac{3}{7}, \frac{4}{9}, \frac{5}{7}$ and $\frac{8}{11}$ (iii) $\frac{1}{10}, \frac{6}{11}, \frac{8}{11}$ and $\frac{3}{5}$
6. Find the greater fraction :
   (i) $\frac{3}{5}$ and $\frac{11}{15}$ (ii) $\frac{4}{5}$ and $\frac{3}{10}$ (iii) $\frac{6}{7}$ and $\frac{5}{9}$
7. Insert one fraction between :
   (i) $\frac{3}{7}$ and $\frac{4}{9}$ (ii) 2 and $\frac{8}{3}$ (iii) $\frac{9}{17}$ and $\frac{6}{13}$
8. Insert three fractions between :
   (i) $\frac{2}{5}$ and $\frac{4}{9}$ (ii) $\frac{1}{2}$ and $\frac{5}{7}$ (iii) $\frac{3}{8}$ and $\frac{6}{11}$
9. Insert two fractions between :
   (i) 1 and $\frac{3}{11}$ (ii) $\frac{5}{9}$ and $\frac{1}{4}$ (iii) $\frac{5}{6}$ and $1\frac{1}{5}$

3.10 OPERATIONS ON FRACTIONS

1. ***Addition and Subtraction :***

   (i) *For like fractions, add* or subtract (as required) *their numerators, keeping the denominator same :*

   $$\therefore\ \frac{1}{8}+\frac{5}{8}=\frac{1+5}{8}=\frac{6}{8}=\frac{3}{4} \text{ and } \frac{\mathbf{9}}{\mathbf{10}}-\frac{\mathbf{3}}{\mathbf{10}}=\frac{9-3}{10}=\frac{6}{10}=\frac{\mathbf{3}}{\mathbf{5}}$$

   (ii) *For unlike fractions,* first of all change given fractions into like fractions and then do the addition or subtraction as above :

   $$\therefore\ \frac{5}{7}-\frac{1}{4}=\frac{5\times4}{7\times4}-\frac{1\times7}{4\times7} \qquad \text{[L.C.M. of 7 and 4 is 28]}$$

   $$=\frac{20}{28}-\frac{7}{28}=\frac{20-7}{28}=\frac{\mathbf{13}}{\mathbf{28}}$$

   or, simply : $\frac{5}{7}-\frac{1}{4}=\frac{5\times4-1\times7}{28}=\frac{20-7}{28}=\frac{\mathbf{13}}{\mathbf{28}}$

   And, $\frac{3}{4}+\frac{2}{5}-\frac{1}{3}=\frac{3\times15}{4\times15}+\frac{2\times12}{5\times12}-\frac{1\times20}{3\times20}$ [L.C.M. of 4, 5 and 3 = 60]

   $$=\frac{45}{60}+\frac{24}{60}-\frac{20}{60}=\frac{45+24-20}{60}=\frac{\mathbf{49}}{\mathbf{60}}$$

   or, simply : $\frac{3}{4}+\frac{2}{5}-\frac{1}{3}=\frac{3\times15+2\times12-1\times20}{60}$

   $$=\frac{45+24-20}{60}=\frac{\mathbf{49}}{\mathbf{60}}.$$

2. ***Multiplication :***

   (i) *To multiply a fraction with an integer, multiply its numerator with the integer.*

   $$\therefore\ 5\times\frac{3}{8}=\frac{5\times3}{8}=\frac{15}{8}=1\frac{7}{8} \text{ and } \frac{4}{15}\times-7=\frac{4\times-7}{15}=\frac{-28}{15}=-1\frac{13}{15}.$$

   (ii) *To multiply two or more fractions, multiply their numerators together and their denominators separately together.*

   $$\therefore\ \frac{3}{5}\times\frac{2}{7}=\frac{3\times2}{5\times7}=\frac{\mathbf{6}}{\mathbf{35}} \text{ and } \frac{3}{8}\times\frac{4}{5}\times\frac{2}{3}=\frac{3\times4\times2}{8\times5\times3}=\frac{\mathbf{1}}{\mathbf{5}}.$$

3. ***Division :***

To divide one quantity (fraction or integer) *by some other quantity (fraction or integer),* multiply the first by the reciprocal of the second.

Reciprocal of 8 = $\frac{1}{8}$, reciprocal of $\frac{1}{-5}=-5$, reciprocal of $\frac{2}{7}=\frac{7}{2}$, reciprocal of $\frac{-12}{25}=\frac{-25}{12}$ and so on.

e.g. (i) $\frac{5}{8}\div2=\frac{5}{8}\times\frac{1}{2}=\frac{\mathbf{5}}{\mathbf{16}}$

(ii) $2\div\frac{5}{8}=2\times\frac{8}{5}=\frac{16}{5}=3\frac{1}{5}$ [Reciprocal of 2 is $\frac{1}{2}$]

(iii) $\frac{7}{10}\div\frac{3}{4}=\frac{7}{10}\times\frac{4}{3}=\frac{28}{30}=\frac{\mathbf{14}}{\mathbf{15}}$ and so on. [Reciprocal of $\frac{5}{8}$ is $\frac{8}{5}$]

3.11 USING "OF"

The word "of" between any two fractions, is to be used as multiplication.

e.g. (i) $\frac{3}{16}$ **of 2** $= \frac{3 \times 2}{16} = \frac{3}{8}$

(ii) $\frac{1}{3}$ **of 18 kg** $= \frac{1 \times 18}{3}$ kg $=$ **6 kg**

(iii) $\frac{3}{4}$ **of ₹ 16** $=$ ₹ $\frac{3 \times 16}{4} =$ ₹ 12 and so on.

3.12 USING "BODMAS" :

The word '**BODMAS**' is the abbreviation formed by taking the initial letters of six operations; '**B**racket', '**O**f', '**D**ivision', '**M**ultiplication', '**A**ddition' and '**S**ubtraction'.

According to the rule of BODMAS, working must be done in the order corresponding to the letters appearing in the word, *i.e.*, first of all the terms inside **B**racket must be simplified; then **O**f must be simplied and then **D**ivision, **M**ultiplication, **A**ddition and finally **S**ubtraction.

e.g. $\left(\frac{1}{3} + \frac{2}{9}\right)$ **of** $\frac{8}{15} \div \frac{4}{9} \times \frac{3}{4} - \frac{1}{2} + 1$

$= \left(\frac{3 + 2}{9}\right)$ of $\frac{8}{15} \div \frac{4}{9} \times \frac{3}{4} - \frac{1}{2} + 1$ **First step (B) :** Simplifying the **Bracket**.

$= \frac{5}{9} \times \frac{8}{15} \div \frac{4}{9} \times \frac{3}{4} - \frac{1}{2} + 1$ **Second step (O) :** Removal of '**Of**'

$= \frac{8}{27} \times \frac{9}{4} \times \frac{3}{4} - \frac{1}{2} + 1$ **Third step (D) : Division**, *i.e.*, multiply by reciprocal.

$= \frac{8 \times 9 \times 3}{27 \times 4 \times 4} - \frac{1}{2} + 1$ **Fourth step (M) : Multiplication.**

$= \frac{1}{2} - \frac{1}{2} + 1$ **Fifth step (A and S) : Addition and Subtraction**

$= 1$ **(Ans.)**

Example 9 :

Evaluate :

(i) $2\frac{1}{4} \div \frac{5}{7} \times 1\frac{1}{3}$

(ii) $\frac{1}{4}$ of $2\frac{2}{7} \div \frac{4}{15}$

Solution :

If required, convert the mixed fraction / fractions into improper fraction / fractions, then apply BODMAS and simplify.

(i) $2\frac{1}{4} \div \frac{5}{7} \times 1\frac{1}{3} = \frac{9}{4} \div \frac{5}{7} \times \frac{4}{3}$

$= \frac{9}{4} \times \frac{7}{5} \times \frac{4}{3} = \frac{9 \times 7 \times 4}{4 \times 5 \times 3} = \frac{21}{5} = 4\frac{1}{5}$ **(Ans.)**

(ii) $\frac{1}{4}$ of $2\frac{2}{7} \div \frac{4}{15} = \frac{1}{4}$ of $\frac{16}{7} \div \frac{4}{15}$ $\left[\because \frac{1}{4} \text{ of } \frac{16}{7} = \frac{1}{4} \times \frac{16}{7} = \frac{4}{7}\right]$

$= \frac{4}{7} \div \frac{4}{15}$

$= \frac{4}{7} \times \frac{15}{4} = \frac{15}{7} = 2\frac{1}{7}$ (Ans.)

Example 10 :

Evaluate :

(i) $\frac{4}{5} \div \frac{7}{15}$ of $\frac{8}{9}$ (ii) $\frac{4}{5} \div \frac{7}{15} \times \frac{8}{9}$ (iii) $\frac{5}{6}$ of $\frac{5}{13} \div \frac{15}{16} \times 1\frac{1}{2}$

Solution :

Remember : BODMAS

(i) $\frac{4}{5} \div \frac{7}{15}$ of $\frac{8}{9} = \frac{4}{5} \div \frac{56}{135}$ $\left[\frac{7}{15} \text{ of } \frac{8}{9} = \frac{56}{135}\right]$

$= \frac{4}{5} \times \frac{135}{56} = \frac{27}{14} = 1\frac{13}{14}$ (Ans.)

(ii) $\frac{4}{5} \div \frac{7}{15} \times \frac{8}{9} = \frac{4}{5} \times \frac{15}{7} \times \frac{8}{9}$ [Division (÷) first]

$= \frac{4 \times 15 \times 8}{5 \times 7 \times 9} = \frac{32}{21} = 1\frac{11}{21}$ (Ans.)

(iii) $\frac{5}{6}$ of $\frac{5}{13} \div \frac{15}{16} \times 1\frac{1}{2} = \frac{25}{78} \div \frac{15}{16} \times \frac{3}{2}$

$= \frac{25}{78} \times \frac{16}{15} \times \frac{3}{2} = \frac{25 \times 16 \times 3}{78 \times 15 \times 2} = \frac{20}{39}$ (Ans.)

EXERCISE 3(C)

1. Reduce to a single fraction :

(i) $\frac{1}{2} + \frac{2}{3}$ (ii) $\frac{3}{5} - \frac{1}{10}$ (iii) $\frac{2}{3} - \frac{1}{6}$

(iv) $1\frac{1}{3} + 2\frac{1}{4}$ (v) $\frac{1}{4} + \frac{5}{6} - \frac{1}{12}$ (vi) $\frac{2}{3} - \frac{3}{5} + 3 - \frac{1}{5}$

(vii) $\frac{2}{3} - \frac{1}{5} + \frac{1}{10}$ (viii) $2\frac{1}{2} + 2\frac{1}{3} - 1\frac{1}{4}$ (ix) $2\frac{5}{8} - 2\frac{1}{6} + 4\frac{3}{4}$

2. Simplify :

(i) $\frac{3}{4} \times 6$ (ii) $\frac{2}{3} \times 15$ (iii) $\frac{3}{4} \times \frac{1}{2}$

(iv) $\frac{9}{12} \times \frac{4}{7}$ (v) $45 \times 2\frac{1}{3}$ (vi) $36 \times 3\frac{1}{4}$

(vii) $2 \div \frac{1}{3}$ (viii) $3 \div \frac{2}{5}$ (ix) $1 \div \frac{3}{5}$

(x) $\frac{1}{3} \div \frac{1}{4}$ (xi) $-\frac{5}{8} \div \frac{3}{4}$ (xii) $3\frac{3}{7} \div 1\frac{1}{14}$

(xiii) $3\frac{3}{4} \times 1\frac{1}{5} \times \frac{20}{21}$

3. Subtract :

(i) 2 from $\frac{2}{3}$ (ii) $\frac{1}{8}$ from $\frac{5}{8}$ (iii) $-\frac{2}{5}$ from $\frac{2}{5}$

(iv) $-\frac{3}{7}$ from $\frac{3}{7}$ (v) 0 from $-\frac{4}{5}$ (vi) $\frac{2}{9}$ from $\frac{4}{5}$

(vii) $-\frac{4}{7}$ from $-\frac{6}{11}$

4. Find the value of :

(i) $\frac{1}{2}$ of 10 kg (ii) $\frac{3}{5}$ of 1 hour (iii) $\frac{4}{7}$ of $2\frac{1}{3}$ kg

(iv) $3\frac{1}{2}$ times of 2 metre (v) $\frac{1}{2}$ of $2\frac{2}{3}$ (vi) $\frac{5}{11}$ of $\frac{4}{5}$ of 22 kg

5. Simplify and reduce to a simple fraction :

(i) $\dfrac{3}{3\frac{3}{4}}$ (ii) $\dfrac{\frac{3}{5}}{7}$ (iii) $\dfrac{3}{\frac{5}{7}}$ (iv) $\dfrac{2\frac{1}{5}}{1\frac{1}{10}}$

(v) $\frac{2}{5}$ of $\frac{6}{11} \times 1\frac{1}{4}$ (vi) $2\frac{1}{4} \div \frac{1}{7} \times \frac{1}{3}$ (vii) $\frac{1}{3} \times 4\frac{2}{3} \div 3\frac{1}{2} \times \frac{1}{2}$

(viii) $\frac{2}{3} \times 1\frac{1}{4} \div \frac{3}{7}$ of $2\frac{5}{8}$ (ix) $0 \div \frac{8}{11}$ (x) $\frac{4}{5} \div \frac{7}{15}$ of $\frac{8}{9}$

(xi) $\frac{4}{5} \div \frac{7}{15} \times \frac{8}{9}$ (xii) $\frac{4}{5}$ of $\frac{7}{15} \div \frac{8}{9}$ (xiii) $\frac{1}{2}$ of $\frac{3}{4} \times \frac{1}{2} \div \frac{2}{3}$

6. A bought $3\frac{3}{4}$ kg of wheat and $2\frac{1}{2}$ kg of rice. Find the total weight of wheat and rice bought.

7. Which is greater, $\frac{3}{5}$ or $\frac{7}{10}$ and by how much ?

8. What number should be added to $8\frac{2}{3}$ to get $12\frac{5}{6}$?

9. What should be subtracted from $8\frac{3}{4}$ to get $2\frac{2}{3}$?

10. A rectangular field is $16\frac{1}{2}$ m long and $12\frac{2}{5}$ m wide. Find the perimeter of the field.

11. Sugar costs ₹ $37\frac{1}{2}$ per kg. Find the cost of $8\frac{3}{4}$ kg sugar.

12. A motor cycle runs $31\frac{1}{4}$ km consuming 1 litre of petrol. How much distance will it run consuming $1\frac{3}{5}$ litre of petrol ?

13. A rectangular park has length = $23\frac{2}{5}$ m and breadth = $16\frac{2}{3}$ m. Find the area of the park.

14. Each of 40 identical boxes weighs $4\frac{4}{5}$ kg. Find the total weight of all the boxes.

15. Out of 24 kg of wheat, $\frac{5}{6}$th of wheat is consumed. Find, how much wheat is still left ?

16. A rod of length $2\frac{2}{5}$ metre is divided into five equal parts. Find the length of each part so obtained.

17. If $A = 3\frac{3}{8}$ and $B = 6\frac{5}{8}$, find : (i) $A \div B$ (ii) $B \div A$.

18. Cost of $3\frac{5}{7}$ litres of oil is ₹ $83\frac{1}{2}$. Find the cost of one litre oil.

19. The product of two numbers is $20\frac{5}{7}$. If one of these numbers is $6\frac{2}{3}$, find the other.

20. By what number should $5\frac{5}{6}$ be multiplied to get $3\frac{1}{3}$?

3.13 USING BRACKETS

The types of brackets used, in general, are :

(i) ( ) are known as *Circular brackets* or *Parentheses* or *simply small brackets.*

(ii) { } are known as *Curly (middle) brackets.*

(iii) [ ] are known as *Square brackets* or *Box brackets or big brackets.*

Sometimes a **bar** is drawn above some terms which we want to treat as a single quantity.

e.g., (i) $\overline{4+5}$ means $(4 + 5) = 9$

(ii) $8 - \overline{3+2} = 8 - 5 = 3$

(iii) $3 + \overline{8-6} = 3 + 2 = 5$ and so on.

This "——" is known as **Bar bracket** or **Vinculum.**

Note : Multiplication sign is often omitted before a bracket and between the brackets.

e.g., (i) $4(9 - 3) = 4 \times (9 - 3) = 4 \times 6 = 24$

(ii) $(2 + 8)(7 - 3) = (2 + 8) \times (7 - 3) = 10 \times 4 = 40$

3.14 REMOVAL OF BRACKETS

The brackets are removed in the order given below :

(i) ―― ; bar or vinculum,

(ii) ( ) ; parentheses,

(iii) { } ; curly brackets,

(iv) [ ] ; square brackets.

Example 11 :

Simplify : $10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - (7 - \overline{6 - 4})\}\right]$

Solution :

$$= 10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - (7 - 2)\}\right] \qquad [\because \overline{6 - 4} = 2]$$

$$= 10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - 5\}\right] \qquad [\because (7 - 2) = 5]$$

$$= 10\frac{1}{2} - \left[8\frac{1}{2} + 1\right] \qquad [\because \{6 - 5\} = 1]$$

$$= 10\frac{1}{2} - 9\frac{1}{2} \qquad \left[\because 8\frac{1}{2} + 1 = 9\frac{1}{2}\right]$$

$$= 1$$

(Ans.)

1. *Whenever there is a positive (+) sign before a bracket, the bracket is removed without any change in the signs of its terms.*
   e.g., $8 + (3 - 1 + 5) = 8 + 3 - 1 + 5 = 16 - 1 = 15$
2. *Whenever there is a negative (–) sign before a bracket, the bracket is removed by changing the signs of all the terms inside the bracket (i.e., by changing every positive sign into negative and every negative sign into positive)*
   e.g., $8 - (3 - 1 + 5) = 8 - 3 + 1 - 5 = 9 - 8 = 1$

EXERCISE 3(D)

Simplify :

1. $6 + \left\{\frac{4}{3} + \left(\frac{3}{4} - \frac{1}{3}\right)\right\}$

2. $8 - \left\{\frac{3}{2} + \left(\frac{3}{5} - \frac{1}{2}\right)\right\}$

3. $\frac{1}{4}\left(\frac{1}{4} + \frac{1}{3}\right) - \frac{2}{5}$

4. $2\frac{3}{4} - \left[3\frac{1}{8} + \left\{5 - \left(4\frac{2}{3} - \frac{11}{12}\right)\right\}\right]$

5. $12\frac{1}{2} - \left[8\frac{1}{2} + \left\{9 - \left(5 - \overline{3 - 2}\right)\right\}\right]$

6. $1\frac{1}{5} + \left\{2\frac{1}{3} - \left(5 + \overline{2 - 3}\right)\right\} - 3\frac{1}{2}$

7. $\left(\frac{1}{2} + \frac{2}{3}\right) \div \left(\frac{3}{4} - \frac{2}{9}\right)$

8. $\frac{6}{5}$ of $\left(3\frac{1}{3} - 2\frac{1}{2}\right) \div \left(2\frac{5}{21} - 2\right)$

9. $10\frac{1}{8}$ of $\frac{4}{5} \div \frac{35}{36}$ of $\frac{20}{49}$

10. $5\frac{3}{4} - \frac{3}{7} \times 15\frac{3}{4} + 2\frac{2}{35} \div 1\frac{11}{25}$

11. $\frac{3}{4}$ of $7\frac{3}{7} - 5\frac{3}{5} \div 3\frac{4}{15}$

3.15 PROBLEMS INVOLVING FRACTIONS

Example 12 :

What fraction is 6 bananas of four dozen bananas ?

Solution :

Here 6 bananas are to be compared with 4 dozens *i.e.*, 4 × 12 = 48 bananas.

∴ **Required fraction** = $\frac{6}{48} = \frac{1}{8}$ (Ans.)

Example 13 :

Write all the natural numbers that lie between 5 and 15 ?

(i) How many of these natural numbers are odd ?

(ii) What fraction of these natural numbers are even ?

Solution :

Since, natural numbers between 5 and 15 are : 6, 7, 8, 9, 10, 11, 12, 13 and 14.

∴ **There are 9 natural numbers between 5 and 15.** (Ans.)

(i) Out of these natural numbers, odd natural numbers are : 7, 9, 11 and 13.

∴ **There are 4 odd natural numbers between 5 and 15.** (Ans.)

(ii) Out of all the given 9 natural numbers, 4 are odd.

∴ Remaining 9 – 4 = 5 numbers are even.

So, the required fraction = $\frac{5}{9}$ (Ans.)

Example 14 :

The monthly income of a man is ₹ 18,000. He gives one-third of it to his wife and one-third of the remaining he spends on his children's education. Find :

(i) the money he gave to his wife.

(ii) the money he spends on his children's education.

(iii) the money still left with him.

Solution :

(i) **The man gives to his wife** = $\frac{1}{3}$ of ₹ 18,000

= $\frac{1}{3}$ × ₹ 18,000 = ₹ 6,000 (Ans.)

(ii) Since, remaining money = ₹ 18,000 – ₹ 6,000 = ₹ 12,000

He spends on his children's education

= $\frac{1}{3}$ × ₹ 12,000 = $\frac{1}{3}$ × ₹ 12,000 = ₹ 4,000 (Ans.)

(iii) **The money still left with the man**

= ₹ 12,000 – ₹ 4,000 = ₹ 8,000 (Ans.)

Example 15 :

Subtract the sum of $\frac{1}{4}$ and $\frac{3}{8}$ from the sum of $\frac{2}{3}, \frac{3}{4}$ and $\frac{7}{12}$.

Solution :

∵ Sum of $\frac{1}{4}$ and $\frac{3}{8} = \frac{1}{4} + \frac{3}{8} = \frac{2+3}{8} = \frac{5}{8}$

And, sum of $\frac{2}{3}, \frac{3}{4}$ and $\frac{7}{12} = \frac{2}{3} + \frac{3}{4} + \frac{7}{12} = \frac{8+9+7}{12} = \frac{24}{12} = 2$

∴ **Required number** $= 2 - \frac{5}{8} = \frac{2}{1} - \frac{5}{8} = \frac{16-5}{8} = \frac{11}{8} = 1\frac{3}{8}$ **(Ans.)**

or, directly, $\left(\frac{2}{3} + \frac{3}{4} + \frac{7}{12}\right) - \left(\frac{1}{4} + \frac{3}{8}\right) = \left(\frac{8+9+7}{12}\right) - \left(\frac{2+3}{8}\right)$

$$= \frac{24}{12} - \frac{5}{8}$$

$$= \frac{48-15}{24} = \frac{33}{24} = \frac{11}{8} = 1\frac{3}{8}$$ **(Ans.)**

Example 16 :

A man spent $\frac{2}{7}$ of his savings and still has ₹ 1,000 left with him. How much were his savings ?

Solution :

The man spent $\frac{2}{7}$ of his money.

∴ He still has $1 - \frac{2}{7} = \frac{5}{7}$ of his savings

Note : In fractions, the whole quantity is always taken as 1.

Since, $\frac{5}{7}$ of his savings = ₹ 1,000

∴ His savings = ₹ 1,000 ÷ $\frac{5}{7}$ = ₹ 1,000 × $\frac{7}{5}$ = ₹ 1,400 **(Ans.)**

Example 17 :

$\frac{4}{7}$ of a pole is in the mud. When $\frac{1}{3}$ of it is pulled out, 250 cm of the pole is still in the mud. What is the full length of the pole ?

Solution :

$\frac{4}{7}$ of the pole − $\frac{1}{3}$ of the pole = 250 cm

⇒ $\left(\frac{4}{7} - \frac{1}{3}\right)$ of the pole = 250 cm

⇒ $\frac{5}{21}$ of the pole = 250 cm $\left[\frac{4}{7} - \frac{1}{3} = \frac{12-7}{21} = \frac{5}{21}\right]$

⇒ **Length of the pole** = 250 × $\frac{21}{5}$ cm = 1050 cm **(Ans.)**

EXERCISE 3(E)

1. A line AB is of length 6 cm. Another line CD is of length 15 cm. What fraction is :
   (i) the length of AB to that of CD ?
   (ii) $\frac{1}{2}$ the length of AB to that of $\frac{1}{3}$ of CD ?
   (iii) $\frac{1}{5}$ of CD to that of AB ?
2. Subtract $\left(\frac{2}{7}-\frac{5}{21}\right)$ from the sum of $\frac{3}{4}$, $\frac{5}{7}$ and $\frac{7}{12}$.
3. From a sack of potatoes weighing 120 kg, a merchant sells portions weighing 6 kg, $5\frac{1}{4}$ kg, $9\frac{1}{2}$ kg and $9\frac{3}{4}$ kg respectively.
   (i) How many kg did he sell ? (ii) How many kg are still left in the sack ?
4. If a boy works for six consecutive days for 8 hours, $7\frac{1}{2}$ hours, $8\frac{1}{4}$ hours, $6\frac{1}{4}$ hours, $6\frac{3}{4}$ hours and 7 hours respectively, how much money will he earn at the rate of ₹ 36 per hour ?
5. A student bought $4\frac{1}{3}$ m of yellow ribbon, $6\frac{1}{6}$ m of red ribbon and $3\frac{2}{9}$ m of blue ribbon for decorating a room. How many metres of ribbon did he buy ?
6. In a business, Ram and Deepak invest $\frac{3}{5}$ and $\frac{2}{5}$ of the total investment. If ₹ 40,000 is the total investment, calculate the amount invested by each.
7. Geeta had 30 problems for home work. She worked out $\frac{2}{3}$ of them. How many problems were still left to be worked out by her ?
8. A picture was marked at ₹ 90. It was sold at $\frac{3}{4}$ of its marked price. What was the sale price ?
9. Mani had sent fifteen parcels of oranges. What was the total weight of the parcels, if each weighed $10\frac{1}{2}$ kg ?
10. A rope is $25\frac{1}{2}$ m long. How many pieces each of $1\frac{1}{2}$ m length can be cut out from it ?
11. The heights of two vertical poles, above the earth's surface, are $14\frac{1}{4}$ m and $22\frac{1}{3}$ m respectively. How much higher is the second pole as compared with the height of the first pole ?
12. Vijay weighed $65\frac{1}{2}$ kg. He gained $1\frac{2}{5}$ kg during the first week, $1\frac{1}{4}$ kg during the second week, but lost $\frac{5}{16}$ kg during the third week. What was his weight after the third week ?

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13. A man spends $\frac{2}{5}$ of his salary on food and $\frac{3}{10}$ on house rent, electricity, etc. What fraction of his salary is still left with him ?

14. A man spends $\frac{2}{5}$ of his salary on food and $\frac{3}{10}$ of the remaining on house rent, electricity, etc. What fraction of his salary is still left with him ?

15. Shyam bought a refrigerator for ₹ 5,000. He paid $\frac{1}{10}$ of the price in cash and the rest in 12 equal monthly instalments. How much had he to pay each month ?

16. A lamp post has half of its length in mud and $\frac{1}{3}$ of its length in water.
    - (i) What fraction of its length is above the water ?
    - (ii) If $3\frac{1}{3}$ m of the lamp post is above the water, find the whole length of the lamp post.

17. I spent $\frac{3}{5}$ of my savings and still have ₹ 2,000 left. What were my savings ?

18. In a school $\frac{4}{5}$ of the children are boys. If the number of girls is 200, find the number of boys.

19. If $\frac{4}{5}$ of an estate is worth ₹ 42,000, find the worth of the whole estate. Also, find the value of $\frac{3}{7}$ of it.

20. After going $\frac{3}{4}$ of my journey, I find that I have covered 16 km. How much journey is still left ?

21. When Krishna travelled 25 km, he found that $\frac{3}{5}$ of his journey was still left. What was the length of the whole journey?

22. From a piece of land, one-third is bought by Rajesh and one-third of remaining is bought by Manoj. If 600 m^2 land is still left unsold, find the total area of the piece of land.

23. A boy spent $\frac{3}{5}$ of his money on buying cloth and $\frac{1}{4}$ of the remaining on buying shoes. If initially he has ₹ 2,400; how much did he spend on shoes ?

24. A boy spent $\frac{3}{5}$ of his money on buying cloth and $\frac{1}{4}$ of his money on buying shoes. If initially he has ₹ 2,400 how much did he spend on shoes.