As per the latest curriculum prepared by the Council for the Indian School Certificate Examinations, New Delhi

Revised

CONCISE Mathematics

– Middle School

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SELINA

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MATHEMATICS CURRICULUM FOR CLASS VIII

TEACHING POINTS	KEY CONCEPTS
1. NUMBER SYSTEM	
Rational Numbers	• Properties of rational numbers. (including identities). Using general form of expression to describe properties • Representation of rational numbers on the number line • Understanding that between any two rational numbers there lies another rational number • Word problems
Exponents and Powers	• Laws of exponents with integral powers • Square and Square roots using factor method and division method for numbers containing (a) no more than total 4 digits and (b) no more than 2 decimal places • Cubes and cube roots (only factor method for numbers containing at most 3 digits)
Playing with numbers	• Writing and understanding a 2 and 3-digit number in generalized form (100a + 10b + c, where a, b, c can be only digit 0-9) and engaging with various puzzles Children to solve and create problems and puzzles. • Deducing the divisibility test rules of 2, 3, 5, 9, 10 for a two or three-digit number expressed in the general form.
Sets	• Union and intersection of sets • Disjoint set • Complement of a set
2. RATIO AND PROPORTION	• Slightly advanced problems involving applications on percentages, profit & loss, overhead expenses, discount, tax. • Difference between simple and compound interest (compounded yearly up to 3 years or half-yearly up to 3 steps only • Direct and inverse variations – Simple and direct word problems • Time and work problems– simple and direct word problems
3. ALGEBRA	• Algebraic Expressions • Multiplication and division of algebraic expression (Coefficient should be integers) • Identities $(a \pm b)^2 = a^2 \pm 2ab + b^2$, $a^2 - b^2 = (a - b)(a + b)$ • Factorisation (simple cases only) as examples the following types $a(x + y)$, $(x \pm y)^2$, $a^2 - b^2$, $(x + a)(x + b)$ • Solving linear equations in one variable in contextual problems involving multiplication and division (word problems) (avoid complex coefficient in the equations)

4. GEOMETRY

Understanding shapes

- Properties of quadrilaterals – Angle Sum property
- Properties of parallelogram (By verification) (i) Opposite sides of a parallelogram are equal, (ii) Opposite angles of a parallelogram are equal, (iii) Diagonals of a parallelogram bisect each other. (iv) Diagonals of a rectangle are equal and bisect each other. (v) Diagonals of a rhombus bisect each other at right angles. (vi) Diagonals of a square are equal and bisect each other at right angles.

Representing 3-D in 2-D

- Identify and match pictures with objects [more complicated e.g. nested, joint 2-D and 3-D shapes (not more than 2)].
- Drawing 2-D representation of 3-D objects (Continued and extended)
- Counting vertices, edges and faces and verifying Euler's relation for 3-D figures with flat faces (cubes, cuboids, tetrahedrons, prisms and pyramids)

Construction of Quadrilaterals

- Given four sides and one diagonal
- Three sides and two diagonals
- Three sides and two included angles
- Two adjacent sides and three angles

5. MENSURATION

- Area of a trapezium and a polygon.
- Surface area of a cube, cuboid, cylinder.
- Concept of volume, measurement of volume using a basic unit, volume of a cube, cuboid and cylinder
- Volume and capacity (measurement of capacity)

6. DATA HANDLING

- Arranging ungrouped data into groups, representation of grouped data through bargraphs, constructing and interpreting bar-graphs.
- Simple Pie charts with reasonable data numbers
- Consolidating and generalising the notion of chance in events like tossing coins, dice etc. Relating it to chance in life events.

Theme – 1
NUMBER
SYSTEM

RATIONAL NUMBERS 1

1.1 INTRODUCTION

We know that :

1. **Natural numbers** = Counting numbers
   = 1, 2, 3, 4, 5, ...
2. **Whole numbers** = 0 (zero) with natural numbers
   = 0, 1, 2, 3, 4, 5, ...
3. **Integers** = Negative of natural numbers together with whole numbers.
   = ..., –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, ...
   Negative of natural numbers + Whole numbers

In class VII, rational numbers were introduced and we did addition, subtraction, multiplication and division of rational numbers. Now we shall be discussing rational numbers and operations on them in detail.

1.2 RATIONAL NUMBER

If p and q both are integers and $q \neq 0$, then $\frac{p}{q}$ is called a rational number.

For example :

1. $\frac{-3}{7}$ is a rational number as –3 and 7 both are integers and $7 \neq 0$.
2. $\frac{15}{22}$ is a rational number as 15 and 22 both are integers and $22 \neq 0$.

Remember :

1. Zero (0) can be written as $\frac{0}{1}, \frac{0}{2}, \frac{0}{5}, \frac{0}{-10}, \frac{0}{15}, \frac{0}{-22}$, etc. In each of these cases denominator is not equal to zero.
   So, zero can be expressed as a fraction with a non-zero denominator.
   ∴ **Zero (0) is a rational number.**
2. Every natural number, every whole number, every integer and every fraction is a rational number.
3. In the rational number $\frac{p}{q}$, where p and q are integers and $q \neq 0$, integer p is called numerator and integer q is called the denominator.

   For example :

   (i) In $\frac{-8}{15}$, numerator = –8 and denominator = 15.

   (ii) If numerator = 5 and denominator = –2, the rational number is $\frac{5}{-2}$.

4. A rational number is positive, if its numerator and denominator have same signs whereas a rational number is negative, if its numerator and denominator have opposite signs.

Thus,

(i) each of $\frac{5}{8}, \frac{-5}{-8}, \frac{-12}{-17}, \frac{15}{19}$, etc. is positive.

(ii) each of $\frac{-5}{8}, \frac{5}{-8}, \frac{12}{-17}, \frac{-15}{19}$, etc. is negative.

5. If m is a non-zero integer and $\frac{p}{q}$ is a rational number, then

$\frac{p}{q} = \frac{p \times m}{q \times m}$ and $\frac{p}{q} = \frac{p \div m}{q \div m}$

Here $\frac{p \times m}{q \times m}$ and $\frac{p \div m}{q \div m}$ are rational numbers each equivalent to $\frac{p}{q}$.

6. Let $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that

$\frac{a}{b} = \frac{c}{d} \Rightarrow a \times d = b \times c$

Conversely, $a \times d = b \times c \Rightarrow \frac{a}{b} = \frac{c}{d}$ or, $\frac{a}{c} = \frac{b}{d}$, *etc*

7. A rational number $\frac{p}{q}$ is said to be in **standard form**, if :

(i) p and q have no common divisor (factor) other than one (1)

and (ii) q is positive.

For example :

(i) $\frac{3}{5}$ is a rational number in standard form.

(ii) The rational number $\frac{3}{-5}$ in standard form is $\frac{-3}{5}$.

(iii) $\frac{-21}{36}$ is not in standard form as 21 and 36 have 3 as a common divisor.

Since, $\frac{-21}{36} = \frac{-7 \times 3}{12 \times 3} = \frac{-7}{12}$

$\therefore \frac{-21}{36}$ in standard form is $\frac{-7}{12}$.

Similarly, $\frac{36}{-63} = \frac{4 \times 9}{-7 \times 9} = \frac{4}{-7} = \frac{-4}{7}$.

$\Rightarrow \frac{36}{-63}$ in standard form is $\frac{-4}{7}$.

1.3 PROPERTIES OF ADDITION OF RATIONAL NUMBERS

1. Closure property

If two rational numbers are added together, the result is always a rational number.

For example :

(i) Addition of rational numbers $\frac{3}{4}$ and $\frac{5}{6}$

$= \frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{9+10}{12} = \frac{19}{12}$, *which is a rational number.*

(ii) Addition of rational numbers $\frac{-3}{8}$ and $\frac{5}{12}$

$= \frac{-3}{8} + \frac{5}{12} = \frac{-9}{24} + \frac{10}{24} = \frac{-9+10}{24} = \frac{1}{24}$, *which is a rational number.*

Thus according to the closure property, if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then their addition $\left(\frac{a}{b}+\frac{c}{d}\right)$ is also a rational number.

We say, set of rational numbers is closed for addition.

2. Commutativity

The addition of any two rational numbers is commutative.

According to commutative property of addition, if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then : $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$.

Consider the rational numbers $\frac{-7}{12}$ and $\frac{5}{8}$.

$\frac{-7}{12} + \frac{5}{8} = \frac{-14}{24} + \frac{15}{24} = \frac{-14+15}{24} = \frac{1}{24}$

and, $\frac{5}{8} + \frac{-7}{12} = \frac{15}{24} + \frac{-14}{24} = \frac{15-14}{24} = \frac{1}{24}$

$\therefore \quad \frac{-7}{12} + \frac{5}{8} = \frac{5}{8} + \frac{-7}{12}$

The same can be verified with any pair of rational numbers.

3. Associativity

The addition of rational numbers is associative.

According to this property, if $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then

$$\frac{a}{b} + \left(\frac{c}{d}+\frac{e}{f}\right) = \left(\frac{a}{b}+\frac{c}{d}\right) + \frac{e}{f}$$

Consider the rational numbers $\frac{2}{3}$, $\frac{-5}{6}$ and $\frac{7}{12}$.

$$\therefore \quad \frac{2}{3}+\left(\frac{-5}{6}+\frac{7}{12}\right) = \frac{2}{3}+\left(\frac{-10}{12}+\frac{7}{12}\right)$$

$$= \frac{2}{3}+\frac{-3}{12} = \frac{8}{12}+\frac{-3}{12} = \frac{8-3}{12} = \frac{5}{12}$$

And,

$$\left(\frac{2}{3}+\frac{-5}{6}\right)+\frac{7}{12} = \left(\frac{4}{6}+\frac{-5}{6}\right)+\frac{7}{12}$$

$$= \frac{-1}{6}+\frac{7}{12} = \frac{-2}{12}+\frac{7}{12} = \frac{-2+7}{12} = \frac{5}{12}$$

$$\therefore \quad \frac{2}{3}+\left(\frac{-5}{6}+\frac{7}{12}\right) = \left(\frac{2}{3}+\frac{-5}{6}\right)+\frac{7}{12}$$

In the same way,

(i) $-\frac{5}{8}+\left(\frac{3}{4}+\frac{-7}{16}\right) = \left(\frac{-5}{8}+\frac{3}{4}\right)+\frac{-7}{16}$

(ii) $\frac{15}{-22}+\left(\frac{-8}{11}+\frac{3}{2}\right) = \left(\frac{15}{-22}+\frac{-8}{11}\right)+\frac{3}{2}$ and so on.

4. Existence of additive identity of rational numbers

Additive identity for rational numbers is zero (0).

When the additive identity is added to any rational number or any rational number added to the additive identity, the rational number remains the same.

$\therefore$ 0 + a rational number = The same rational number + 0

= The rational number itself

For example :

(i) $0+\frac{-3}{5} = -\frac{3}{5}+0 = -\frac{3}{5}$.   (ii) $0+\frac{7}{8} = \frac{7}{8}+0 = \frac{7}{8}$ and so on.

Consider a rational number $\frac{a}{b}$, then $\frac{a}{b}+0 = \frac{a}{b} = 0+\frac{a}{b}$

The rational number 0 is called the identity element for addition of rational numbers.

5. Existence of additive inverse of a rational number

The negative of a rational number is called its additive inverse.

(i) The additive inverse of $\frac{3}{5} = -\frac{3}{5}$.

(ii) The additive inverse of $\frac{-5}{8} = \frac{5}{8}$ and so on.

The sum of a rational number and its additive inverse = Additive identity

i.e. Any rational number + its additive inverse = 0, the additive identity

$\Rightarrow \quad \frac{3}{5} + \left(-\frac{3}{5}\right) = 0, \qquad \left(-\frac{5}{8}\right) + \frac{5}{8} = 0$

$\left(\frac{7}{-8}\right) + \frac{7}{8} = 0, \qquad \frac{8}{15} + \left(-\frac{8}{15}\right) = 0$ and so on.

Example 1 :

Add each pair of rational numbers, given below, and show that their addition (sum) is also a rational number :

(i) $\frac{7}{15}$ and $\frac{3}{5}$ (ii) $\frac{2}{5}$ and 2

(iii) $\frac{3}{8}$ and $\frac{-5}{12}$ (iv) $\frac{7}{-15}$ and $\frac{2}{-3}$

(v) $\frac{5}{-13}$ and $\frac{11}{26}$

Solution :

(i) $\frac{7}{15} + \frac{3}{5} = \frac{7}{15} + \frac{3\times3}{5\times3}$ [∵ L.C.M. of 15 and 5 = 15]

$= \frac{7}{15} + \frac{9}{15}$

$= \frac{7+9}{15} = \frac{16}{15}$, *which is a rational number.*

(ii) $\frac{2}{5} + 2 = \frac{2}{5} + \frac{2}{1}$ [∵ L.C.M. of 5 and 1 = 5]

$= \frac{2}{5} + \frac{2\times5}{1\times5}$

$= \frac{2}{5} + \frac{10}{5}$

$= \frac{2+10}{5} = \frac{12}{5}$, *which is a rational number.*

(iii) $\frac{3}{8} + \frac{-5}{12} = \frac{3\times3}{8\times3} + \frac{-5\times2}{12\times2}$ [∵ L.C.M. of 8 and 12 = 24]

$= \frac{9}{24} + \frac{-10}{24} = \frac{9-10}{24} = \frac{-1}{24}$, *which is a rational number.*

(iv) $\frac{7}{-15} + \frac{2}{-3} = \frac{-7}{15} + \frac{-2}{3}$

$= \frac{-7}{15} + \frac{-2\times5}{3\times5}$ [∵ L.C.M. of 15 and 3 = 15]

$= \frac{-7}{15} + \frac{-10}{15} = \frac{-7-10}{15} = \frac{-17}{15}$, *which is a rational number.*

(v) $\frac{5}{-13} + \frac{11}{26} = \frac{-5}{13} + \frac{11}{26}$

$= \frac{-5\times2}{13\times2} + \frac{11}{26}$ [∵ L.C.M. of 13 and 26 = 26]

$= \frac{-10}{26} + \frac{11}{26} = \frac{-10+11}{26} = \frac{1}{26}$, *which is a rational numbers.*

> The examples, given above, show that the addition of two rational numbers is always a rational number. Thus, it verifies the closure property of addition of rational numbers.

Example 2 :

Evaluate :

(i) $\frac{3}{4} + \frac{5}{6} + \frac{-1}{4} + \frac{-7}{6}$ (ii) $\frac{9}{-10} + \frac{4}{15} + \frac{-3}{20} + \frac{-3}{10} + \frac{8}{15} + \frac{9}{-20}$

Solution :

> Re-arrange the given rational numbers and group them in such a way that each group has rational numbers with same denominator.

(i) $\frac{3}{4} + \frac{5}{6} + \frac{-1}{4} + \frac{-7}{6} = \left(\frac{3}{4} + \frac{-1}{4}\right) + \left(\frac{5}{6} + \frac{-7}{6}\right)$

$= \frac{3-1}{4} + \frac{5-7}{6}$

$= \frac{2}{4} + \frac{-2}{6}$

$= \frac{1}{2} - \frac{1}{3}$

$= \frac{1\times3}{2\times3} - \frac{1\times2}{3\times2}$ [∵ L.C.M. of 2 and 3 = 6]

$= \frac{3}{6} - \frac{2}{6} = \frac{3-2}{6} = \frac{1}{6}$ **(Ans.)**

(ii) $\frac{9}{-10} + \frac{4}{15} + \frac{-3}{20} + \frac{-3}{10} + \frac{8}{15} + \frac{9}{-20}$

$= \left(\frac{-9}{10} + \frac{-3}{10}\right) + \left(\frac{4}{15} + \frac{8}{15}\right) + \left(\frac{-3}{20} + \frac{-9}{20}\right)$

$= \frac{-9-3}{10} + \frac{4+8}{15} + \frac{-3-9}{20}$

$= \frac{-12}{10} + \frac{12}{15} + \frac{-12}{20}$

$= -\frac{6}{5} + \frac{4}{5} - \frac{3}{5} = \frac{-6+4-3}{5} = \frac{-5}{5} = -1$ **(Ans.)**

Example 3 :

Use rational numbers $\frac{4}{9}$ and $\frac{-7}{12}$ to verify the commutative proeprty for the addition of rational numbers.

Solution :

Show that : $\frac{4}{9} + \frac{-7}{12} = \frac{-7}{12} + \frac{4}{9}$.

$$\because \quad \frac{4}{9} + \frac{-7}{12} = \frac{4\times4}{9\times4} + \frac{-7\times3}{12\times3} \qquad [\because \text{ L.C.M. of 9 and 12} = 36]$$

$$= \frac{16}{36} - \frac{21}{36} = \frac{16-21}{36} = \frac{-5}{36}$$

And $$\frac{-7}{12} + \frac{4}{9} = \frac{-7\times3}{12\times3} + \frac{4\times4}{9\times4}$$

$$= \frac{-21}{36} + \frac{16}{36} = \frac{-21+16}{36} = \frac{-5}{36}$$

$$\therefore \quad \frac{4}{9} + \frac{-7}{12} = \frac{-7}{12} + \frac{4}{9}$$

This verifies the commutative property for the addition of rational numbers.

Example 4 :

Use rational numbers $\frac{-4}{5}$, $\frac{7}{10}$ and $\frac{11}{-20}$ to verify the associative property of the addition of rational numbers.

Solution :

Show that : $\frac{-4}{5} + \left(\frac{7}{10} + \frac{11}{-20}\right) = \left(\frac{-4}{5} + \frac{7}{10}\right) + \frac{11}{-20}$.

$$\because \quad \frac{-4}{5} + \left(\frac{7}{10} + \frac{11}{-20}\right) = \frac{-4}{5} + \left(\frac{7}{10} + \frac{-11}{20}\right)$$

$$= \frac{-4}{5} + \left(\frac{7\times2}{10\times2} + \frac{-11}{20}\right) \qquad [\because \text{ L.C.M. of 10 and 20} = 20]$$

$$= \frac{-4}{5} + \left(\frac{14}{20} + \frac{-11}{20}\right)$$

$$= \frac{-4}{5} + \left(\frac{14-11}{20}\right)$$

$$= \frac{-4}{5} + \frac{3}{20}$$

$$= \frac{-4\times4}{5\times4} + \frac{3}{20} = \frac{-16}{20} + \frac{3}{20} = \frac{-16+3}{20} = \frac{-13}{20}$$

And, $\left(\frac{-4}{5}+\frac{7}{10}\right)+\frac{11}{-20}=\left(\frac{-4\times2}{5\times2}+\frac{7}{10}\right)+\frac{-11}{20}$ [$\because$ L.C.M. of 5 and 10 = 10]

$$=\left(\frac{-8}{10}+\frac{7}{10}\right)+\frac{-11}{20}$$

$$=\frac{-8+7}{10}+\frac{-11}{20}$$

$$=\frac{-1}{10}+\frac{-11}{20}$$

$$=\frac{-1\times2}{10\times2}+\frac{-11}{20}$$

$$=\frac{-2}{20}+\frac{-11}{20}=\frac{-2-11}{20}=\frac{-13}{20}$$

$$\therefore \quad \frac{-4}{5}+\left(\frac{7}{10}+\frac{11}{-20}\right)=\left(\frac{-4}{5}+\frac{7}{10}\right)+\frac{11}{-20}$$

This verifies associative property of the addition of rational numbers.

Example 5 :

Write the additive inverse of :

(i) $\frac{3}{8}$ (ii) $\frac{-8}{15}$ (iii) $\frac{4}{-13}$ (iv) $\frac{-6}{-11}$

Solution :

The additive inverse of $\frac{a}{b}$ is $-\frac{a}{b}$ and the additive inverse of $-\frac{a}{b}$ is $\frac{a}{b}$.

(i) **The additive inverse of $\frac{3}{8}$ is $-\frac{3}{8}$.** (Ans.)

(ii) **The additive inverse of $\frac{-8}{15}=\frac{8}{15}$.** (Ans.)

(iii) **The additive inverse of $\frac{4}{-13}=\frac{4}{13}$.** (Ans.)

(iv) $\because \frac{-6}{-11}=\frac{6}{11}$

$\therefore$ **The additive inverse of $\frac{-6}{-11}$**

= The additive inverse of $\frac{6}{11}=\frac{-6}{11}$. (Ans.)

EXERCISE 1(A)

1. Add, each pair of rational numbers, given below, and show that their addition (sum) is also a rational number :

(i) $\frac{-5}{8}$ and $\frac{3}{8}$ (ii) $\frac{-8}{13}$ and $\frac{-4}{13}$

(iii) $\frac{6}{11}$ and $\frac{-9}{11}$ (iv) $\frac{5}{-26}$ and $\frac{8}{39}$

(v) $\frac{5}{-6}$ and $\frac{2}{3}$ (vi) -2 and $\frac{2}{5}$

(vii) $\frac{9}{-4}$ and $\frac{-3}{8}$ (viii) $\frac{7}{-18}$ and $\frac{8}{27}$

2. Evaluate :

(i) $\frac{5}{9}+\frac{-7}{6}$ (ii) $4+\frac{3}{-5}$

(iii) $\frac{1}{-15}+\frac{5}{-12}$ (iv) $\frac{5}{9}+\frac{3}{-4}$

(v) $\frac{-8}{9}+\frac{-5}{12}$ (vi) $0+\frac{-2}{7}$

(vii) $\frac{5}{-11}+0$ (viii) $2+\frac{-3}{5}$

(ix) $\frac{4}{-9}+1$

3. Evaluate :

(i) $\frac{3}{7}+\frac{-4}{9}+\frac{-11}{7}+\frac{7}{9}$

(ii) $\frac{2}{3}+\frac{-4}{5}+\frac{1}{3}+\frac{2}{5}$

(iii) $\frac{4}{7}+0+\frac{-8}{9}+\frac{-13}{7}+\frac{17}{9}$

(iv) $\frac{3}{8}+\frac{-5}{12}+\frac{3}{7}+\frac{3}{12}+\frac{-5}{8}+\frac{-2}{7}$

4. For each pair of rational numbers, verify commutative property of additon of rational numbers :

(i) $\frac{-8}{7}$ and $\frac{5}{14}$ (ii) $\frac{5}{9}$ and $\frac{5}{-12}$

(iii) $\frac{-4}{5}$ and $\frac{-13}{-15}$ (iv) $\frac{2}{-5}$ and $\frac{11}{-15}$

(v) 3 and $\frac{-2}{7}$ (vi) -2 and $\frac{3}{-5}$

5. For each set of rational numbers, given below, verify the associative property of addition of rational numbers :

(i) $\frac{1}{2}, \frac{2}{3}$ and $-\frac{1}{6}$

(ii) $\frac{-2}{5}, \frac{4}{15}$ and $\frac{-7}{10}$

(iii) $\frac{-7}{9}, \frac{2}{-3}$ and $\frac{-5}{18}$

(iv) $-1, \frac{5}{6}$ and $\frac{-2}{3}$

6. Write the additive inverse (negative) of :

(i) $\frac{-3}{8}$ (ii) $\frac{4}{-9}$

(iii) $\frac{-7}{5}$ (iv) $\frac{-4}{-13}$

(v) 0 (vi) -2

(vii) 1 (viii) $-\frac{1}{3}$

(ix) $\frac{-3}{1}$

7. Fill in the blanks :

(i) Additive inverse of $\frac{-5}{-12}$ = ...

(ii) $\frac{-5}{-12}$ + its additive inverse = ...

(iii) If $\frac{a}{b}$ is additive inverse of $\frac{-c}{d}$, then $\frac{-c}{d}$ is additive inverse of ....

And so $\frac{a}{b}+\frac{(-c)}{d}=\frac{(-c)}{d}+\frac{a}{b}$ = ...

8. State, true or false :

(i) $\frac{7}{9}=\frac{7+5}{9+5}$ (ii) $\frac{7}{9}=\frac{7-5}{9-5}$

(iii) $\frac{7}{9}=\frac{7\times 5}{9\times 5}$ (iv) $\frac{7}{9}=\frac{7\div 5}{9\div 5}$

(v) $\frac{-5}{-12}$ is a negative rational number

(vi) $\frac{-13}{25}$ is smaller than $\frac{-25}{13}$.

1.4 SUBTRACTION OF RATIONAL NUMBERS

(i) Subtraction of $\frac{3}{4}$ from $\frac{5}{6}$

$= \frac{5}{6} - \frac{3}{4}$

$= \frac{5\times2}{6\times2} - \frac{3\times3}{4\times3}$ [$\because$ L.C.M. of 6 and 4 = 12]

$= \frac{10}{12} - \frac{9}{12} = \frac{10-9}{12} = \frac{1}{12}$

(ii) Subtraction of $\frac{-5}{8}$ from $\frac{-7}{12}$

$= \frac{-7}{12} - \left(\frac{-5}{8}\right)$

$= \frac{-7}{12} + \frac{5}{8}$

$= \frac{-7\times2}{12\times2} + \frac{5\times3}{8\times3}$ [$\because$ L.C.M. of 12 and 8 = 24]

$= \frac{-14}{24} + \frac{15}{24} = \frac{-14+15}{24} = \frac{1}{24}$

Example 6 :

The sum of two rational numbers is $\frac{-5}{8}$. If one of these numbers is $\frac{-7}{12}$, find the other.

Solution :

$\because$ The sum of two rational numbers = $\frac{-5}{8}$

and, one of the numbers = $\frac{-7}{12}$

$\therefore$ The other rational number

$= \frac{-5}{8} - \left(\frac{-7}{12}\right)$

$= -\frac{5}{8} + \frac{7}{12}$

$= \frac{-5\times3}{8\times3} + \frac{7\times2}{12\times2}$ [$\because$ L.C.M. of 8 and 12 = 24]

$= \frac{-15}{24} + \frac{14}{24}$

$= \frac{-15+14}{24} = \frac{-1}{24}$ (Ans.)

Example 7 :

What should be added to $-\frac{3}{8}$ to get $\frac{5}{6}$?

Solution :

Required rational number

$$= \frac{5}{6} - \left(-\frac{3}{8}\right)$$

$$= \frac{5}{6} + \frac{3}{8}$$

$$= \frac{5\times4}{6\times4} + \frac{3\times3}{8\times3}$$

$$= \frac{20}{24} + \frac{9}{24} = \frac{29}{24} = 1\frac{5}{24} \text{ (Ans.)}$$

Alternative method :

Let x be added

$$\therefore -\frac{3}{8} + x = \frac{5}{6}$$

$$\Rightarrow \quad x = \frac{5}{6} + \frac{3}{8}$$

$$= \frac{5\times4}{6\times4} + \frac{3\times3}{8\times3}$$

$$= \frac{20}{24} + \frac{9}{24} = \frac{29}{24} = 1\frac{5}{24} \quad \text{(Ans.)}$$

Example 8 :

What should be subtracted from $-\frac{3}{8}$ to get $\frac{5}{6}$?

Solution :

Required rational number

$$= -\frac{3}{8} - \frac{5}{6}$$

$$= \frac{-3\times3}{8\times3} - \frac{5\times4}{6\times4}$$

$$= \frac{-9}{24} - \frac{20}{24}$$

$$= \frac{-9-20}{24}$$

$$= \frac{-29}{24} \quad \text{(Ans.)}$$

Alternative method :

Let x be subtracted

$$\therefore \quad -\frac{3}{8} - x = \frac{5}{6}$$

$$\Rightarrow \quad -\frac{3}{8} - \frac{5}{6} = x \quad i.e. \quad x = -\frac{3}{8} - \frac{5}{6}$$

$$\Rightarrow \quad x = \frac{-3\times3}{8\times3} - \frac{5\times4}{6\times4}$$

$$= \frac{-9}{24} - \frac{20}{24}$$

$$= \frac{-9-20}{24} = \frac{-29}{24} \quad \text{(Ans.)}$$

1.5 PROPERTIES OF SUBTRACTION OF RATIONAL NUMBERS

1. Closure property

(i) $\frac{3}{5} - \frac{5}{10} = \frac{3\times2}{5\times2} - \frac{5}{10}$

$= \frac{6}{10} - \frac{5}{10} = \frac{6-5}{10} = \frac{1}{10}$, *which is a rational number.*

(ii) $\frac{7}{12} - \frac{5}{18} = \frac{7\times3}{12\times3} - \frac{5\times2}{18\times2}$

$= \frac{21}{36} - \frac{10}{36} = \frac{21-10}{36} = \frac{11}{36}$, *which is a rational number.*

Thus according to the closure property, if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers then $\frac{a}{b}-\frac{c}{d}$ is also a rational number. And so is $\frac{c}{d}-\frac{a}{b}$.

2. Commutativity

The subtraction of rational numbers is not commutative.

Consider the rational numbers $\frac{-7}{12}$ and $\frac{5}{8}$.

$$\frac{-7}{12}-\frac{5}{8}=\frac{-7\times2}{12\times2}-\frac{5\times3}{8\times3}$$
$$=\frac{-14}{24}-\frac{15}{24}$$
$$=\frac{-14-15}{24}=\frac{-29}{24}$$

and,
$$\frac{5}{8}-\left(\frac{-7}{12}\right)=\frac{5}{8}+\frac{7}{12}$$
$$=\frac{5\times3}{8\times3}+\frac{7\times2}{12\times2}$$
$$=\frac{15}{24}+\frac{14}{24}=\frac{15+14}{24}=\frac{29}{24}$$

$$\Rightarrow \quad \frac{-7}{12}-\frac{5}{8}\neq\frac{5}{8}-\left(\frac{-7}{12}\right)$$

Thus, if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then : $\frac{a}{b}-\frac{c}{d}\neq\frac{c}{d}-\frac{a}{b}$.
Hence, the subtraction of rational numbers is not commutative.

3. Associativity

The subtraction of rational numbers is not associative. i.e. if $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then

$$\frac{a}{b}-\left(\frac{c}{d}-\frac{e}{f}\right)\neq\left(\frac{a}{b}-\frac{c}{d}\right)-\frac{e}{f}$$

Consider the rational numbers $\frac{2}{3}$, $\frac{-5}{6}$ and $\frac{7}{12}$.

$$\therefore \quad \frac{2}{3}-\left(\frac{-5}{6}-\frac{7}{12}\right)=\frac{2}{3}-\left(\frac{-5\times2}{6\times2}-\frac{7}{12}\right)$$
$$=\frac{2}{3}-\left(\frac{-10}{12}-\frac{7}{12}\right)$$
$$=\frac{2}{3}-\left(\frac{-17}{12}\right)$$

$$= \frac{2}{3} + \frac{17}{12}$$

$$= \frac{2\times4}{12} + \frac{17}{12} = \frac{8+17}{12} = \frac{25}{12}$$

$$\left(\frac{2}{3} - \frac{-5}{6}\right) - \frac{7}{12} = \left(\frac{2\times2}{3\times2} - \frac{-5}{6}\right) - \frac{7}{12}$$

$$= \left(\frac{4}{6} + \frac{5}{6}\right) - \frac{7}{12}$$

$$= \frac{9}{6} - \frac{7}{12}$$

$$= \frac{9\times2}{6\times2} - \frac{7}{12}$$

$$= \frac{18}{12} - \frac{7}{12} = \frac{11}{12}$$

$$\Rightarrow \quad \frac{2}{3} - \left(\frac{-5}{6} - \frac{7}{12}\right) \neq \left(\frac{2}{3} - \frac{-5}{6}\right) - \frac{7}{12}$$

In the same way,

(i) $3 - \left(\frac{8}{9} - \frac{4}{7}\right) \neq \left(3 - \frac{8}{9}\right) - \frac{4}{7}$

(ii) $\frac{5}{8} - \left(\frac{7}{12} - \frac{9}{17}\right) \neq \left(\frac{5}{8} - \frac{7}{12}\right) - \frac{9}{17}$ and so on.

4. Existence of identity

For a rational number $\frac{a}{b}$,

$\frac{a}{b} - 0 = \frac{a}{b}$, but $0 - \frac{a}{b} \neq \frac{a}{b}$

$\therefore$ Subtraction has only right identity as $\frac{a}{b} - 0 = \frac{a}{b}$, $\frac{5}{8} - 0 = \frac{5}{8}$, $-4 - 0 = -4$ and so on.

And so we say subtraction has no identity.

5. Existence of inverse

Inverse for subtraction does not exist.

EXERCISE 1(B)

1. Evaluate :

(i) $\frac{2}{3} - \frac{4}{5}$ (ii) $\frac{-4}{9} - \frac{2}{-3}$

(iii) $-1 - \frac{4}{9}$ (iv) $\frac{-2}{7} - \frac{3}{-14}$

(v) $\frac{-5}{18} - \frac{-2}{9}$ (vi) $\frac{5}{21} - \frac{-13}{42}$

2. Subtract :

(i) $\frac{5}{8}$ from $\frac{-3}{8}$ (ii) $\frac{-8}{11}$ from $\frac{4}{11}$

(iii) $\frac{4}{9}$ from $\frac{-5}{9}$ (iv) $\frac{1}{4}$ from $\frac{-3}{8}$

(v) $\frac{-5}{8}$ from $\frac{-13}{16}$ (vi) $\frac{-9}{22}$ from $\frac{5}{33}$

3. The sum of two rational numbers is $\frac{9}{20}$. If one of them is $\frac{2}{5}$, find the other.

4. The sum of two rational numbers is $\frac{-2}{3}$. If one of them is $\frac{-8}{15}$, find the other.

5. The sum of the two rational numbers is –6. If one of them is $\frac{-8}{5}$, find the other.

6. Which rational number should be added to $\frac{-7}{8}$ to get $\frac{5}{9}$?

7. Which rational number should be added to $\frac{-5}{9}$ to get $\frac{-2}{3}$?

8. Which rational number should be subtracted from $\frac{-5}{6}$ to get $\frac{4}{9}$?

9. (i) What should be subtracted from –2 to get $\frac{3}{8}$?

   (ii) What should be added to –2 to get $\frac{3}{8}$?

10. Evaluate :

   (i) $\frac{3}{7}+\frac{-4}{9}-\frac{-11}{7}-\frac{7}{9}$

   (ii) $\frac{2}{3}+\frac{-4}{5}-\frac{1}{3}-\frac{2}{5}$

   (iii) $\frac{4}{7}-\frac{-8}{9}-\frac{-13}{7}+\frac{17}{9}$

1.6 MULTIPLICATION OF RATIONAL NUMBERS

Multiplication (product) **of two rational numbers**

$$= \frac{\text{Product of their numerators}}{\text{Product of their denominators}}$$

Thus, if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then

$$\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$$

(i) $\frac{2}{5}\times\frac{3}{4} = \frac{2\times 3}{5\times 4} = \frac{1\times 3}{5\times 2} = \frac{3}{10}$

(ii) $\frac{-3}{5}\times\frac{4}{7} = \frac{-3\times 4}{5\times 7} = \frac{-12}{35}$

(iii) $\left(\frac{-15}{8}\right)\times\frac{-4}{5} = \frac{(-15)\times(-4)}{8\times 5}$

$= \frac{15\times 4}{8\times 5} = \frac{3\times 1}{2\times 1} = \frac{3}{2}$

1.7 PROPERTIES OF MULTIPLICATION OF RATIONAL NUMBERS

1. **Closure property**

If any two rational numbers are multiplied together, the result is always a rational number.

For example :

(i) Multiplication of $\frac{3}{4}$ and $\frac{5}{6}$

$$= \frac{3}{4} \times \frac{5}{6} = \frac{3 \times 5}{4 \times 6} = \frac{1 \times 5}{4 \times 2} = \frac{5}{8}, \text{ which is a rational number.}$$

(ii) Multiplication of $\frac{-3}{8}$ and $\frac{5}{12}$

$$= \frac{-3}{8} \times \frac{5}{12} = \frac{-3 \times 5}{8 \times 12} = \frac{-1 \times 5}{8 \times 4} = \frac{-5}{32}, \text{ which is a rational number.}$$

2. Commutativity

The multiplication of any two rational numbers is commutative.

Consider the rational numbers $\frac{-7}{12}$ and $\frac{5}{8}$.

$$\frac{-7}{12} \times \frac{5}{8} = \frac{-7 \times 5}{12 \times 8} = \frac{-35}{96}$$

and, $$\frac{5}{8} \times \frac{-7}{12} = \frac{5 \times (-7)}{8 \times 12} = \frac{-35}{96}$$

$\therefore$ $$\frac{-7}{12} \times \frac{5}{8} = \frac{5}{8} \times \frac{-7}{12}$$

The same can be verified with any pair of rational numbers.

According to commutative property of multiplication, if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then : $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$.

3. Associativity

According to this property, if $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then

$$\frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f}$$

Consider the rational numbers $\frac{2}{3}$, $\frac{-5}{6}$ and $\frac{7}{12}$.

$\therefore$ $$\frac{2}{3} \times \left(\frac{-5}{6} \times \frac{7}{12}\right) = \frac{2}{3} \times \left(\frac{-5 \times 7}{6 \times 12}\right)$$

$$= \frac{2}{3} \times \frac{-35}{72} = \frac{2 \times -35}{3 \times 72} = \frac{1 \times -35}{3 \times 36} = \frac{-35}{108}$$

And, $$\left(\frac{2}{3} \times \frac{-5}{6}\right) \times \frac{7}{12} = \left(\frac{2 \times -5}{3 \times 6}\right) \times \frac{7}{12}$$

$$= \frac{-5}{9} \times \frac{7}{12} = \frac{-5 \times 7}{9 \times 12} = \frac{-35}{108}$$

$\Rightarrow$ $$\frac{2}{3} \times \left(\frac{-5}{6} \times \frac{7}{12}\right) = \left(\frac{2}{3} \times \frac{-5}{6}\right) \times \frac{7}{12}$$

In the same way,

(i) $-\frac{5}{8} \times \left(\frac{3}{4} \times \frac{-7}{16}\right) = \left(\frac{-5}{8} \times \frac{3}{4}\right) \times \frac{-7}{16}$

(ii) $\frac{15}{-22} \times \left(\frac{-8}{11} \times \frac{3}{2}\right) = \left(\frac{15}{-22} \times \frac{-8}{11}\right) \times \frac{3}{2}$ and so on.

4. Existence of multiplicative identity of rational numbers

When multiplicative identity is multiplied with any rational number or any rational number is multiplied with multiplicative identity, the rational number remains the same.

> Multiplicative identity for rational numbers is one (1).
>
> $\therefore$ 1 × any rational number = The same rational number × 1
>
> = The number itself
>
> That is, for national number $\frac{a}{b}$, $1 \times \frac{a}{b} = \frac{a}{b} \times 1 = \frac{a}{b}$.

For example :

(i) $\frac{5}{7} \times 1 = 1 \times \frac{5}{7} = \frac{5}{7}$.

(ii) $\frac{-20}{47} \times 1 = 1 \times \frac{-20}{47} = \frac{-20}{47}$ and so on.

5. Existence of multiplicative inverse of rational numbers

The reciprocal of a rational number is called its multiplicative inverse.

(i) The multiplicative inverse of $\frac{3}{5}$ = reciprocal of $\frac{3}{5} = \frac{5}{3}$.

(ii) The multiplicative inverse of $\frac{-5}{8}$ = reciprocal of $\frac{-5}{8} = \frac{8}{-5}$ and so on.

> Rational number 0 (zero) does not have its multiplicative inverse.

The product of a rational number and its multiplicative inverse = multiplicative identity

i.e. A rational number × its multiplicative inverse = 1, *the multiplicative identity*

$\Rightarrow \quad \frac{3}{5} \times \frac{5}{3} = 1, \quad \left(\frac{-5}{8}\right) \times \left(\frac{8}{-5}\right) = 1$

$\left(\frac{7}{-8}\right) \times \left(\frac{-8}{7}\right) = 1, \quad \frac{-8}{-15} \times \frac{-15}{-8} = 1$ and so on.

> For a rational number $\frac{a}{b}$
>
> its multiplicative inverse is $\frac{b}{a}$ such that : $\frac{a}{b} \times \frac{b}{a} = 1$
>
> **1 is the multiplicative inverse of itself and so is –1.**

6. Distributivity of multiplication over addition

The multiplication of rational numbers is distributive over their addition/subtraction.

If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then

(i) $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} + \frac{a}{b} \times \frac{e}{f}$

(ii) $\frac{a}{b} \times \left(\frac{c}{d} - \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} - \frac{a}{b} \times \frac{e}{f}$

Consider any three numbers; $\frac{3}{4}$, $\frac{-4}{5}$ and $\frac{5}{6}$.

$$\therefore \quad \frac{3}{4} \times \left(\frac{-4}{5} + \frac{5}{6}\right) = \frac{3}{4} \times \left(\frac{-4 \times 6 + 5 \times 5}{30}\right)$$

$$= \frac{3}{4} \times \left(\frac{-24 + 25}{30}\right) = \frac{3}{4} \times \frac{1}{30} = \frac{3 \times 1}{4 \times 30} = \frac{1}{40}$$

And, $$\frac{3}{4} \times \frac{-4}{5} + \frac{3}{4} \times \frac{5}{6} = \frac{3 \times -4}{4 \times 5} + \frac{3 \times 5}{4 \times 6}$$

$$= \frac{-3}{5} + \frac{5}{8}$$

$$= \frac{-3 \times 8 + 5 \times 5}{40} = \frac{-24 + 25}{40} = \frac{1}{40}$$

$$\therefore \quad \frac{3}{4} \times \left(\frac{-4}{5} + \frac{5}{6}\right) = \frac{3}{4} \times \frac{-4}{5} + \frac{3}{4} \times \frac{5}{6}$$

In the same way,

$$\frac{3}{4} \times \left(\frac{-4}{5} - \frac{5}{6}\right) = \frac{3}{4} \times \left(\frac{-24 - 25}{30}\right)$$

$$= \frac{3}{4} \times \frac{-49}{30} = \frac{3 \times -49}{4 \times 30} = \frac{1 \times -49}{4 \times 10} = -\frac{49}{40}$$

And, $$\frac{3}{4} \times \frac{-4}{5} - \frac{3}{4} \times \frac{5}{6} = \frac{3 \times -4}{4 \times 5} - \frac{3 \times 5}{4 \times 6}$$

$$= \frac{-3}{5} - \frac{5}{8}$$

$$= \frac{-3 \times 8 - 5 \times 5}{40} = \frac{-24 - 25}{40} = -\frac{49}{40}$$

$$\therefore \quad \frac{3}{4} \times \left(\frac{-4}{5} - \frac{5}{6}\right) = \frac{3}{4} \times \frac{-4}{5} - \frac{3}{4} \times \frac{5}{6}$$

EXERCISE 1(C)

1. Evaluate :

(i) $\frac{-14}{5}\times\frac{-6}{7}$ (ii) $\frac{7}{6}\times\frac{-18}{91}$

(iii) $\frac{-125}{72}\times\frac{9}{-5}$ (iv) $\frac{-11}{9}\times\frac{-51}{-44}$

(v) $-\frac{16}{5}\times\frac{20}{8}$

2. Multiply :

(i) $\frac{5}{6}$ and $\frac{8}{9}$ (ii) $\frac{2}{7}$ and $\frac{-14}{9}$

(iii) $\frac{-7}{8}$ and 4 (iv) $\frac{36}{-7}$ and $\frac{-9}{28}$

(v) $\frac{-7}{10}$ and $\frac{-8}{15}$ (vi) $\frac{3}{-2}$ and $\frac{-7}{3}$

3. Evaluate :

(i) $\left(\frac{2}{-3}\times\frac{5}{4}\right)+\left(\frac{5}{9}\times\frac{3}{-10}\right)$

(ii) $\left(2\times\frac{1}{4}\right)-\left(\frac{-18}{7}\times\frac{-7}{15}\right)$

(iii) $\left(-5\times\frac{2}{15}\right)-\left(-6\times\frac{2}{9}\right)$

(iv) $\left(\frac{8}{5}\times\frac{-3}{2}\right)+\left(\frac{-3}{10}\times\frac{9}{16}\right)$

4. Multiply each rational number, given below, by one (1) :

(i) $\frac{7}{-5}$ (ii) $\frac{-3}{-4}$

(iii) 0 (iv) $\frac{-8}{13}$

(v) $\frac{-6}{-7}$

5. For each pair of rational numbers, given below, verify that the multiplication is commutative :

(i) $\frac{-1}{5}$ and $\frac{2}{9}$ (ii) $\frac{5}{-3}$ and $\frac{13}{-11}$

(iii) 3 and $\frac{-8}{9}$ (iv) 0 and $\frac{-12}{17}$

6. Write the reciprocal (multiplicative inverse) of each rational number, given below :

(i) 5 (ii) −3

(iii) $\frac{5}{11}$ (iv) $\frac{-7}{-8}$

(v) $\frac{-8}{-7}$ (vi) $\frac{15}{-17}$

7. Find the reciprocal (multiplicative inverse) of :

(i) $\frac{3}{5}\times\frac{2}{3}$ (ii) $\frac{-8}{3}\times\frac{13}{-7}$

(iii) $\frac{-3}{5}\times\frac{-1}{13}$

8. Verify that $(x+y)\times z = x\times z + y\times z$, if

(i) $x=\frac{4}{5}$, $y=-\frac{-2}{3}$ and $z=-4$

(ii) $x=2$, $y=\frac{4}{5}$ and $z=\frac{3}{-10}$

9. Verify that $x\times(y-z) = x\times y - x\times z$, if

(i) $x=\frac{4}{5}$, $y=-\frac{7}{4}$ and $z=3$

(ii) $x=\frac{3}{4}$, $y=\frac{8}{9}$ and $z=-5$

10. Name the multiplication property of rational numbers shown below :

(i) $\frac{3}{5}\times\frac{-8}{9}=\frac{-8}{9}\times\frac{3}{5}$

(ii) $\frac{-3}{4}\times\left(\frac{5}{7}\times\frac{-8}{15}\right)=\left(\frac{-3}{4}\times\frac{5}{7}\right)\times\frac{-8}{15}$

(iii) $\frac{4}{5}\times\left(\frac{3}{-8}+\frac{-4}{7}\right)=\frac{4}{5}\times\frac{3}{-8}+\frac{4}{5}\times\frac{-4}{7}$

(iv) $\frac{-7}{5}\times\frac{5}{-7}=1$

(v) $\frac{8}{-9}\times 1 = 1\times\frac{8}{-9}=\frac{8}{-9}$

11. Fill in the blanks :

(i) The product of two positive rational numbers is always ....

(ii) The product of two negative rational numbers is always ....

(iii) If two rational numbers have opposite signs then their product is always ....

(iv) The reciprocal of a positive rational number is .... and the reciprocal of a negative rational number is ....

(v) Rational number 0 has .... reciprocal.

(vi) The product of a rational number and its reciprocal is ....

(vii) The numbers .... and .... are their own reciprocals.

(viii) If m is reciprocal of n, then the reciprocal of n is ....

1.8 DIVISION OF RATIONAL NUMBERS

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers such that $\frac{c}{d} \neq 0$, then :

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times (\text{reciprocal of } \frac{c}{d})$$

$$= \frac{a}{b} \times \frac{d}{c}$$

- If $\frac{c}{d} = 0$, the division $\frac{a}{b} \div \frac{c}{d}$ is *not defined.*

$\Rightarrow$ **division by 0 is not defined.**

- If m and n are two rational numbers such that $n \neq 0$, then m divided by n is the rational number obtained on multiplying m by the reciprocal of n. Thus $m \div n = m \times \frac{1}{n}$.
- Division is the inverse of multiplication.

For example :

(i) $\frac{3}{4}$ **divided by** $\frac{5}{12} = \frac{3}{4} \div \frac{5}{12} = \frac{3}{4} \times \frac{12}{5} = \frac{9}{5} = 1\frac{4}{5}$

(ii) $\frac{-6}{7}$ **divided by** $\frac{4}{21} = -\frac{6}{7} \div \frac{4}{21} = -\frac{6}{7} \times \frac{21}{4} = -\frac{9}{2}$

(iii) $-\frac{16}{27} \div \frac{-8}{9} = -\frac{16}{27} \times \frac{9}{-8} = \frac{2}{3}$

1.9 PROPERTIES OF DIVISION OF RATIONAL NUMBERS

1. Closure property

If a rational number is divided by some non-zero rational number, the result is always a rational number.

1. If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers and $\frac{c}{d} \neq 0$, then $\left(\frac{a}{b} \div \frac{c}{d}\right)$ is also a rational number.

2. For every rational number $\frac{a}{b}$

(i) $\frac{a}{b} \div 1 = \frac{a}{b}$ (ii) $\frac{a}{b} \div \frac{a}{b} = 1$

For example :

(i) $\frac{3}{4}$ and $\frac{5}{8}$ are two rational numbers. Since, $\frac{5}{8}$ is not equal to zero ($\frac{5}{8} \neq 0$), therefore $\frac{3}{4} \div \frac{5}{8}$ is a rational number.

$\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \times \frac{8}{5} = \frac{6}{5}$, a rational number.

(ii) 0 and $\frac{5}{2}$ are two rational numbers and $\frac{5}{2} \neq 0$,

then $0 \div \frac{5}{2} = 0 \times \frac{2}{5} = 0$, *a rational number.*

0 is a rational number.

(iii) 4 and $\frac{2}{3}$ are two rational numbers such that $\frac{2}{3} \neq 0$, then $4 \div \frac{2}{3} = 4 \times \frac{3}{2} = 6$, *a rational number.*

2. Commutativity

Division of two different rational numbers is not commutative.

i.e., If $\frac{a}{b}$ and $\frac{c}{d}$ are two non-zero rational numbers then : $\frac{a}{b} \div \frac{c}{d} \neq \frac{c}{d} \div \frac{a}{b}$.

The same can be verified with any pair of rational numbers.

3. Associativity

Division of rational numbers is not associative.

i.e. If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are rational numbers such that $\frac{c}{d} \neq 0$ and $\frac{e}{f} \neq 0$; then

$$\frac{a}{b} \div \left(\frac{c}{d} \div \frac{e}{f}\right) \neq \left(\frac{a}{b} \div \frac{c}{d}\right) \div \frac{e}{f}$$

4. Identity for division of rational numbers does not exist.

5. Inverse for division of rational numbers does not exist.

Example 9 :

The product of two rational numbers is $\frac{8}{9}$. If one of them is $-\frac{5}{6}$, find the other.

Solution :

$\because$ The product of two rational numbers is $= \frac{8}{9}$ and one of them is $-\frac{5}{6}$

$\therefore$ **The other number** $= \frac{8}{9} \div \left(-\frac{5}{6}\right)$

$$= \frac{8}{9} \times \left(-\frac{6}{5}\right) = -\frac{8 \times 6}{9 \times 5} = -\frac{8 \times 2}{3 \times 5} = -\frac{16}{15}$$

Example 10 :

By what number must $-\frac{5}{8}$ be multiplied, so that the product is $\frac{3}{4}$.

Solution :

$\because$ The product of two numbers is $\frac{3}{4}$ and one of them is $-\frac{5}{8}$

$\therefore$ **The other number** $= \frac{3}{4} \div \left(-\frac{5}{8}\right)$

$$= \frac{3}{4} \times \left(-\frac{8}{5}\right) = -\frac{24}{20} = -\frac{6}{5}$$

EXERCISE 1(D)

1. Evaluate :

   (i) $1 \div \frac{1}{3}$ (ii) $3 \div \frac{3}{5}$

   (iii) $-\frac{5}{12} \div \frac{1}{16}$ (iv) $-\frac{21}{16} \div \left(\frac{-7}{8}\right)$

   (v) $0 \div \left(-\frac{4}{7}\right)$ (vi) $\frac{8}{-5} \div \frac{24}{25}$

   (vii) $-\frac{3}{4} \div (-9)$ (viii) $\frac{3}{4} \div \left(-\frac{5}{12}\right)$

   (ix) $-5 \div \left(-\frac{10}{11}\right)$ (x) $\frac{-7}{11} \div \left(\frac{-3}{44}\right)$

2. Divide :

   (i) 3 by $\frac{1}{3}$ (ii) -2 by $\left(-\frac{1}{2}\right)$

   (iii) 0 by $\frac{7}{-9}$ (iv) $\frac{-5}{8}$ by $\frac{1}{4}$

   (v) $-\frac{3}{4}$ by $-\frac{9}{16}$

3. The product of two rational numbers is -2. If one of them is $\frac{4}{7}$, find the other.

4. The product of two numbers is $-\frac{4}{9}$. If one of them is $\frac{-2}{27}$, find the other.

5. m and n are two rational numbers such that $m \times n = -\frac{25}{9}$.

   (i) if $m = \frac{5}{3}$, find n, (ii) if $n = -\frac{10}{9}$, find m.

6. By what number must $-\frac{3}{4}$ be multiplied so that the product is $-\frac{9}{16}$?

7. By what number should $\frac{-8}{13}$ be multiplied to get 16 ?

8. If $3\frac{1}{2}$ litres of milk costs ₹ 49, find the cost of one litre of milk ?

9. Cost of $3\frac{2}{5}$ metre of cloth is ₹ $88\frac{1}{2}$. What is the cost of 1 metre of cloth ?

10. Divide the sum of $\frac{3}{7}$ and $\frac{-5}{14}$ by $-\frac{1}{2}$.

11. Find $(m + n) \div (m - n)$, if :

   (i) $m = \frac{2}{3}$ and $n = \frac{3}{2}$

   (ii) $m = \frac{3}{4}$ and $n = \frac{4}{3}$

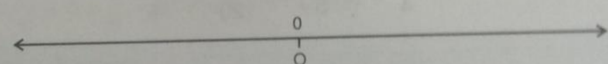
   (iii) $m = \frac{4}{5}$ and $n = -\frac{3}{10}$

12. The product of two rational numbers is −5. If one of these numbers is $\frac{-7}{15}$, find the other.

13. Divide the sum of $\frac{5}{8}$ and $\frac{-11}{12}$ by the difference of $\frac{3}{7}$ and $\frac{5}{14}$.

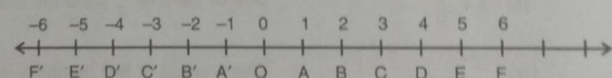
1.10 REPRESENTATION OF RATIONAL NUMBERS ON THE NUMBER LINE

Draw a line of suitable length. Nearly at the middle of this line, mark a point O that represents number zero (0).



Starting from O, mark points on this line at equal distances both on right as well as on left of O. Let A, B, C, D, etc. be the points on the right side of O and A′, B′, C′, D′, etc. be the points on the left side of O so that :

OA = AB = BC = CD = ... = OA′ = A′B′ = B′C′ = C′D′ = ...



If OA = 1 unit

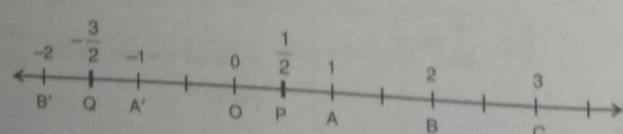
⇒ A, B, C, D, etc. represent integers 1, 2, 3, 4, ... respectively and A′, B′, C′, D′ etc. represent integers −1, −2, −3, −4, ... respectively.

Example 11 :

Represent $\frac{1}{2}$ and $-\frac{3}{2}$ on a number line.

Solution :

Draw a number line as shown below :



In this number line

OA = AB = ... = OA′ = A′B′ = ... = 1 unit

Since, denominator of each given rational number is 2; divide each of OA, AB, BC, OA′, A′B′, etc. into two equal parts.

To represent $\frac{1}{2}$, move one step towards right side of O to reach point P as shown.

$\because$ OA = 1 unit, therefore OP = $\frac{1}{2}$ unit and so **P represents $\frac{1}{2}$**. **(Ans.)**

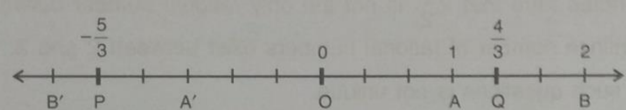
In the same way, to represent $-\frac{3}{2}$, move 3 steps towards the left side of O to reach point Q. Clearly Q, = $-\frac{3}{2}$. **(Ans.)**

Example 12 :

Represent $-\frac{5}{3}$ and $\frac{4}{3}$ on a number line.

Solution :

Draw a number line as shown below :



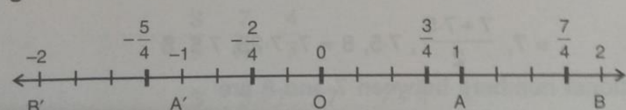
Since, the denominator of each given rational number is 3; divide each of OA, AB, OA′, A′B′, etc. into three equal parts.

To mark $-\frac{5}{3}$, move 5 parts to the left of O to reach point P, therefore P = $-\frac{5}{3}$.

To mark $\frac{4}{3}$, move 4 parts to the right of O to reach point Q, therefore Q = $\frac{4}{3}$.

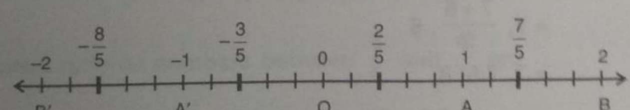
The following number lines will make the concept more clear

1.



- Since, each of the rational numbers $-\frac{5}{4}, -\frac{2}{4}, \frac{3}{4}$ and $\frac{7}{4}$ has 4 in its denominator, divide OA = AB = OA′ = A′B′, etc. into 4 equal parts.

2.



- Since, each of the rational numbers $-\frac{8}{5}, -\frac{3}{5}, \frac{2}{5}$ and $\frac{7}{5}$ has 5 in its denominator, divide OA = AB = OA′ = A′B′, etc. into 5 equal parts.

1.11 INSERTING RATIONAL NUMBERS BETWEEN TWO GIVEN RATIONAL NUMBERS

First method :

If a and b are two rational numbers, then $\frac{a+b}{2}$ is also a rational number and its value lies between a and b.

(i) If $a < b \Rightarrow a < \frac{a+b}{2} < b$ i.e. $5 < 8 \Rightarrow 5 < \frac{5+8}{2} < 8$ i.e. $\mathbf{5 < 6{\cdot}5 < 8}$

(ii) If $a > b \Rightarrow a > \frac{a+b}{2} > b$ i.e. $8 > 5 \Rightarrow 8 > \frac{8+5}{2} > 5$ i.e. $\mathbf{8 > 6{\cdot}5 > 5}$

Example 13 :

Insert one rational number between 2 and 3.

Solution :

The rational number between 2 and 3 $= \frac{2+3}{2} = \frac{5}{2} = 2\frac{1}{2}$

It must be noted here that $2\frac{1}{2}$ is not the only rational number between 2 and 3. Infact, an infinite number of rational numbers exist between 2 and 3. Solution to such questions is not unique.

Example 14 :

Insert two rational numbers between 7 and 8.

Solution :

Given numbers = 7 and 8

$= 7, \frac{7+8}{2}, 8$ [Inserting one rational number between 7 and 8

$= 7, 7{\cdot}5, 8$

$= 7, \frac{7+7{\cdot}5}{2}, 7{\cdot}5, 8 = 7, 7{\cdot}25, 7{\cdot}5, 8$

$\therefore$ **Required rational numbers between 7 and 8 are 7·25 and 7·5** (Ans.)

Alternative method :

Given numbers = 7 and 8

$= 7, \frac{7+8}{2}, 8$

$= 7, 7{\cdot}5, 8$

$= 7, 7{\cdot}5, \frac{7{\cdot}5+8}{2}, 8 = 7, 7{\cdot}5, 7{\cdot}75, 8$

$\therefore$ **Required rational numbers between 7 and 8 are 7·5 and 7·75** (Ans.)

Example 15 :

Insert three rational numbers between 3 and 4.

Solution :

Given numbers = 3 and 4

$$= 3, \frac{3+4}{2}, 4$$

$$= 3, 3{\cdot}5, 4$$

$$= 3, \frac{3+3{\cdot}5}{2}, 3{\cdot}5, \frac{3{\cdot}5+4}{2}, 4 = 3, 3{\cdot}25, 3{\cdot}5, 3{\cdot}75, 4$$

∴ **Required rational numbers between 3 and 4 are**

3·25, 3·5 and 3·75

Second method :

For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a+c}{b+d}$ is also a rational number with its value lying between $\frac{a}{b}$ and $\frac{c}{d}$.

Example 16 :

Find three rational numbers between $\frac{3}{5}$ and $\frac{4}{7}$.

Solution :

Given numbers = $\frac{3}{5}$ and $\frac{4}{7}$

$$= \frac{3}{5}, \frac{3+4}{5+7}, \frac{4}{7}$$

$$= \frac{3}{5}, \frac{7}{12}, \frac{4}{7}$$

$$= \frac{3}{5}, \frac{3+7}{5+12}, \frac{7}{12}, \frac{7+4}{12+7}, \frac{4}{7}$$

$$= \frac{3}{5}, \frac{10}{17}, \frac{7}{12}, \frac{11}{19}, \frac{4}{7}$$

∴ **Required rational numbers between $\frac{3}{5}$ and $\frac{4}{7}$ are**

$\frac{10}{17}, \frac{7}{12}$ **and** $\frac{11}{17}$

1.12 METHOD OF FINDING LARGE NUMBER OF RATIONAL NUMBERS BETWEEN TWO GIVEN RATIONAL NUMBERS.

Example 17 :

Insert five rational numbers between $\frac{3}{4}$ and $\frac{7}{8}$.

Solution :

Steps : 1. Find L.C.M. of denominators. L.C.M. of denominators 4 and 8 is 8.

2. Make denominator of each given rational number equal to 8 (the L.C.M.)

$$\frac{3}{4} = \frac{3\times2}{4\times2} = \frac{6}{8} \text{ and } \frac{7}{8} = \frac{7}{8}$$

3. Since, five rational numbers are required, multiply the numerator and the denominator of each rational number (obtained in step 2) by 5 + 1 = 6.

$$\therefore \frac{6}{8} = \frac{6\times6}{8\times6} = \frac{36}{48} \text{ and } \frac{7}{8} = \frac{7\times6}{8\times6} = \frac{42}{48}$$

Now every rational number with denominator 48 and numerator between 36 and 42 will have its value between the given rational numbers $\frac{3}{4}$ and $\frac{7}{8}$.

$\Rightarrow$ **Required rational numbers between $\frac{3}{4}$ and $\frac{7}{8}$ are**

$$= \frac{37}{48}, \frac{38}{48}, \frac{39}{48}, \frac{40}{48} \text{ and } \frac{41}{48}$$

$$= \frac{37}{48}, \frac{19}{24}, \frac{13}{16}, \frac{5}{6} \text{ and } \frac{41}{48}$$ **(Ans.)**

Example 18 :

Insert 7 rational numbers between $\frac{5}{6}$ and $\frac{7}{9}$.

Solution :

Step 1 : L.C.M. of denominators 6 and 9 = 18.

Step 2 : Make denominator of each given rational number equal to 18 (the L.C.M.)

$$\frac{5}{6} = \frac{5\times3}{6\times3} = \frac{15}{18} \text{ and } \frac{7}{9} = \frac{7\times2}{9\times2} = \frac{14}{18}$$

Step 3 : Since, seven rational numbers are required between $\frac{5}{6}$ and $\frac{7}{9}$; multiply the numerator and the denominator of each rational number (obtained in step 2) by 7 + 1 = 8

$$\frac{15}{18} = \frac{15\times8}{18\times8} = \frac{120}{144} \text{ and } \frac{14}{18} = \frac{14\times8}{18\times8} = \frac{112}{144}$$

⇒ **Required rational numbers between $\frac{5}{6}$ and $\frac{7}{9}$ are :**

$$\frac{119}{144}, \frac{118}{144}, \frac{117}{144}, \frac{116}{144}, \frac{115}{144}, \frac{114}{144} \text{ and } \frac{113}{144}$$

$$= \frac{119}{144}, \frac{59}{72}, \frac{13}{16}, \frac{29}{36}, \frac{115}{144}, \frac{19}{24} \text{ and } \frac{113}{144} \quad \textbf{(Ans.)}$$

EXERCISE 1(E)

1. Draw a number line and mark $\frac{3}{4}, \frac{7}{4}, \frac{-3}{4}$ and $\frac{-7}{4}$ on it.
2. On a number line mark the points $\frac{2}{3}, \frac{-8}{3}, \frac{7}{3}, \frac{-2}{3}$ and -2.
3. Insert one rational number between
   (i) 7 and 8 (ii) 3·5 and 5
   (iii) 2 and 3·2 (iv) 4·2 and 3·6
   (v) $\frac{1}{2}$ and 2
4. Insert two rational numbers between
   (i) 6 and 7 (ii) 4·8 and 6
   (iii) 2·7 and 6·3
5. Insert three rational numbers between
   (i) 3 and 4 (ii) 10 and 12
6. Insert five rational numbers between $\frac{3}{5}$ and $\frac{2}{3}$.
7. Insert six rational numbers between $\frac{5}{6}$ and $\frac{8}{9}$.
8. Insert seven rational numbers between 2 and 3.

EXPONENTS
(Powers)
2

2.1 REVIEW

Exponent	If x is a real number and n is an integer, we know : $x \times x \times x \times x \times x \times$ .... n times $= x^n$ where x^n is called an **exponential expression** with **base** x and **exponent** (or index, or power) **n.** x^n is read as 'x raised to the power n'.

2.2 LAWS OF EXPONENTS (FOR INTEGRAL POWERS)

1. **Product Law :** $a^m \times a^n = a^{m+n}$

$\Rightarrow$ $3^3 \times 3^5 = 3^{3+5} = 3^8$; $5^8 \times 5^5 = 5^{8+5} = 5^{13}$;

$7^2 \times 7^4 = 7^{2+4} = 7^6$; $2^{-5} \times 2^8 = 2^{-5+8} = 2^3$ and so on.

2. **Quotient Law :** $\dfrac{a^m}{a^n} = a^{m-n}$, if m > n

and $\dfrac{a^m}{a^n} = \dfrac{1}{a^{n-m}}$, if n > m

$\Rightarrow$ $\dfrac{2^{12}}{2^7} = 2^{12-7} = 2^5$; $\dfrac{2^6}{2^{13}} = \dfrac{1}{2^{13-6}} = \dfrac{1}{2^7}$;

$\dfrac{5^{12}}{5^{-3}} = 5^{12+3} = 5^{15}$; $\dfrac{3^{-6}}{3^3} = \dfrac{1}{3^{3+6}} = \dfrac{1}{3^9}$ and so on.

3. **Power Law :** $(a^m)^n = a^{mn}$

$\Rightarrow$ $(3^5)^2 = 3^{5\times2} = 3^{10}$; $(5^6)^{-3} = 5^{6\times-3} = 5^{-18}$

$(7^{-2})^3 = 7^{-2\times3} = 7^{-6}$; $(5^{-3})^{-2} = 5^{-3\times-2} = 5^6$ and so on.

$(-2)^3 = -2 \times -2 \times -2 = -8$,

$(-2)^4 = -2 \times -2 \times -2 \times -2 = 16$,

$(-2)^5 = -2 \times -2 \times -2 \times -2 \times -2 = -32$,

$(-2)^6 = -2 \times -2 \times -2 \times -2 \times -2 \times -2 = 64$ and so on.

Thus

(i) If n is **even**, $(-2)^n$ is **positive**.

(ii) If n is **odd**, $(-2)^n$ is **negative**.

In general, $(-a)^n = a^n$, if n is even

and, $(-a)^n = -a^n$, if n is odd

2.3 NEGATIVE INTEGRAL EXPONENT

For any non-zero rational number a

$a^{-n} = \frac{1}{a^n}$ and $a^n = \frac{1}{a^{-n}}$

i.e. a^{-n} and a^n are reciprocal of each other.

Thus, $5^{-3} = \frac{1}{5^3}$, $2^{-5} = \frac{1}{2^5}$,

$\left(\frac{2}{3}\right)^{-5} = \frac{1}{\left(\frac{2}{3}\right)^5} = \left(\frac{3}{2}\right)^5$, $\left(\frac{5}{8}\right)^{-6} = \frac{1}{\left(\frac{5}{8}\right)^6} = \left(\frac{8}{5}\right)^6$,

$7^3 = \frac{1}{7^{-3}}$, $11^5 = \frac{1}{11^{-5}}$ and so on.

Also

1. $(-2)^{-5} = \frac{1}{(-2)^5} = \frac{1}{-2^5} = \frac{1}{-2\times2\times2\times2\times2} = \frac{1}{-32} = -\frac{1}{32}$

2. $\left(\frac{4}{3}\right)^{-3} = \left(\frac{3}{4}\right)^3 = \frac{3}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{27}{64}$

3. $\left(-\frac{2}{3}\right)^{-4} = \left(-\frac{3}{2}\right)^4 = \left(\frac{3}{2}\right)^4 = \frac{3}{2}\times\frac{3}{2}\times\frac{3}{2}\times\frac{3}{2} = \frac{81}{16}$

4. $\frac{1}{5^{-3}} = 5^3 = 5\times5\times5 = 125$ and so on.

Example 1 :

Evaluate and express as a rational number of the form $\frac{m}{n}$:

(i) $\left(\frac{3}{5}\right)^{-2}\times\left(\frac{4}{5}\right)^{-3}$ (ii) $\left(-\frac{2}{3}\right)^{-4}\times\left(-\frac{3}{5}\right)^{2}$

Solution :

(i) $\left(\frac{3}{5}\right)^{-2}\times\left(\frac{4}{5}\right)^{-3} = \left(\frac{5}{3}\right)^{2}\times\left(\frac{5}{4}\right)^{3}$

$= \frac{5^2}{3^2}\times\frac{5^3}{4^3} = \frac{25\times125}{9\times64} = \frac{3125}{576}$ **(Ans.)**

(ii) $\left(-\frac{2}{3}\right)^{-4}\times\left(-\frac{3}{5}\right)^{2} = \left(-\frac{3}{2}\right)^{4}\times\left(-\frac{3}{5}\right)^{2}$

$= \left(\frac{3}{2}\right)^{4}\times\left(\frac{3}{5}\right)^{2} = \frac{81}{16}\times\frac{9}{25} = \frac{729}{400}$ **(Ans.)**

Example 2 :

Evaluate :

(i) $(2^{-1} \div 5^{-1})^2 \times \left(\frac{-5}{8}\right)^{-1}$ (ii) $(5^{-1} \times 3^{-1})^{-1} \div 6^{-1}$

(iii) $(4^{-1} + 8^{-1}) \div \left(\frac{2}{3}\right)^{-1}$

Solution :

(i) $(2^{-1} \div 5^{-1})^2 \times \left(\frac{-5}{8}\right)^{-1} = \left(\frac{1}{2} \div \frac{1}{5}\right)^2 \times \left(\frac{8}{-5}\right)^1$

$= \left(\frac{1}{2} \times \frac{5}{1}\right)^2 \times \frac{8}{-5}$

$= \left(\frac{5}{2}\right)^2 \times \frac{8}{-5} = \frac{25}{4} \times \frac{8}{-5} = \frac{5 \times 2}{-1} = \mathbf{-10}$ (Ans.)

(ii) $(5^{-1} \times 3^{-1})^{-1} \div 6^{-1} = \left(\frac{1}{5} \times \frac{1}{3}\right)^{-1} \div \frac{1}{6}$

$= \left(\frac{1}{15}\right)^{-1} \div \frac{1}{6} = \frac{15}{1} \times \frac{6}{1} = \mathbf{90}$ (Ans.)

(iii) $(4^{-1} + 8^{-1}) \div \left(\frac{2}{3}\right)^{-1} = \left(\frac{1}{4} + \frac{1}{8}\right) \div \left(\frac{3}{2}\right)$

$= \left(\frac{2+1}{8}\right) \times \frac{2}{3} = \frac{3}{8} \times \frac{2}{3} = \mathbf{\frac{1}{4}}$ (Ans.)

Example 3 :

Evaluate : $\left\{\left(\frac{-3}{2}\right)^{-3}\right\}^2$

Solution :

$\left\{\left(\frac{-3}{2}\right)^{-3}\right\}^2 = \left(-\frac{3}{2}\right)^{-3 \times 2}$

$= \left(-\frac{3}{2}\right)^{-6}$

$= \left(-\frac{2}{3}\right)^6 = \left(\frac{2}{3}\right)^6 = \frac{2^6}{3^6} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3 \times 3} = \mathbf{\frac{64}{729}}$ (Ans.)

Example 4 :

Evaluate : $\left(\frac{1}{3}\right)^{-3} + \left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{5}\right)^{-2}$.

Solution :

$$\left(\frac{1}{3}\right)^{-3} + \left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{5}\right)^{-2} = \left(\frac{3}{1}\right)^{3} + \left(\frac{4}{1}\right)^{3} - \left(\frac{5}{1}\right)^{2}$$

$$= 3^3 + 4^3 - 5^2$$

$$= 27 + 64 - 25 = 66 \qquad \textbf{(Ans.)}$$

Example 5 :

If $3^{3x-1} \div 9 = 27$, find the value of x.

Solution :

$$3^{3x-1} \div 9 = 27 \Rightarrow 3^{3x-1} \times \frac{1}{9} = 27$$

$$\Rightarrow 3^{3x-1} \times \frac{1}{3^2} = 3^3$$

$$\Rightarrow 3^{3x-1-2} = 3^3$$

$$\Rightarrow 3^{3x-3} = 3^3$$

$$\Rightarrow 3x - 3 = 3$$

i.e. $3x = 6$ and $x = 2$ **(Ans.)**

$$3^{3x-1} \times \frac{1}{9} = 27$$

$$\Rightarrow 3^{3x-1} = 27 \times 9$$

$$= 3 \times 3 \times 3 \times 3 \times 3 = 3^5$$

$$\Rightarrow 3x - 1 = 5$$

i.e. $x = \frac{6}{3} = 2$ **(Ans.)**

EXERCISE 2(A)

1. Evaluate :

(i) $(3^{-1} \times 9^{-1}) \div 3^{-2}$ (ii) $(3^{-1} \times 4^{-1}) \div 6^{-1}$

(iii) $(2^{-1} + 3^{-1})^3$ (iv) $(3^{-1} \div 4^{-1})^2$

(v) $(2^2 + 3^2) \times \left(\frac{1}{2}\right)^2$

(vi) $(5^2 - 3^2) \times \left(\frac{2}{3}\right)^{-3}$

(vii) $\left[\left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{3}\right)^{-3}\right] \div \left(\frac{1}{6}\right)^{-3}$

(viii) $\left[\left(-\frac{3}{4}\right)^{-2}\right]^2$

(ix) $\left\{\left(\frac{3}{5}\right)^{-2}\right\}^{-2}$

(x) $(5^{-1} \times 3^{-1}) \div 6^{-1}$

2. If $1125 = 3^m \times 5^n$; find m and n.

3. Find x, if $9 \times 3^x = (27)^{2x-3}$

2.4 MORE ABOUT EXPONENTS

1. $(a \times b)^n = a^n \times b^n$

e.g. $(a^5 \times b^{-3})^4 = (a^5)^4 \times (b^{-3})^4 = a^{20} \times b^{-12}$ and $(3^4 \times 5^{-3})^{-2} = 3^{-8} \times 5^6$

2. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

e.g. $\left(\frac{a^{-3}}{b^4}\right)^6 = \frac{(a^{-3})^6}{(b^4)^6} = \frac{a^{-18}}{b^{24}}$ and $\left(\frac{5^7}{3^{-4}}\right)^{-3} = \frac{5^{-21}}{3^{12}}$

3. $a^0 = 1$; if $a \neq 0$

i.e. any non–zero number raised to the power zero is always equal to one (1).

e.g. $5^0 = 1, 7^0 = 1, (-8)^0 = 1, (2^{-5})^0 = 1$ and so on.

4. $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$; if $a \neq 0$

e.g. $2^{-3} = \frac{1}{2^3}, \frac{1}{5^{-7}} = 5^7, \frac{2^{-3}}{3^{-5}} = \frac{3^5}{2^3}$ and so on.

5. $\sqrt[n]{a} = a^{\frac{1}{n}}$ and $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

e.g. $\sqrt{5} = 5^{\frac{1}{2}}$ $\quad \sqrt[6]{5^7} = 5^{\frac{7}{6}}$ $\quad \sqrt[3]{a^2 \times b^4} = a^{\frac{2}{3}} \times b^{\frac{4}{3}}$, etc.

Also remember that :

(i) $(-a)^m = a^m$; *if m is even* and (ii) $(-a)^m = -a^m$; *if m is odd.*

e.g. $(-5)^4 = -5 \times -5 \times -5 \times -5 = 5^4$ and $(-5)^3 = -5 \times -5 \times -5 = -5^3$

Example 6 :

Evaluate :

(i) $4^{\frac{3}{2}} \times 125^{\frac{-2}{3}}$ (ii) $\left(\frac{8}{27}\right)^{\frac{2}{3}} \div (32)^{\frac{-2}{5}}$

(iii) $-2^4 - \left(\sqrt{3}\right)^0 \times (-2)^6 \div 4$

Solution :

(i) $4^{\frac{3}{2}} \times 125^{\frac{-2}{3}} = (2^2)^{\frac{3}{2}} \times (5^3)^{\frac{-2}{3}}$ $\quad [4 = 2 \times 2 = 2^2,\ 125 = 5 \times 5 \times 5 = 5^3]$

$= 2^3 \times 5^{-2}$ $\quad \left[2 \times \frac{3}{2} = 3 \text{ and } 3 \times \frac{-2}{3} = -2\right]$

$= \frac{8}{5^2}$ $\quad \left[2^3 = 2 \times 2 \times 2 = 8 \text{ and } 5^{-2} = \frac{1}{5^2}\right]$

$= \frac{8}{25}$

(Ans.)

(ii) $\left(\frac{8}{27}\right)^{\frac{2}{3}} \div (32)^{\frac{-2}{5}} = \left(\frac{2}{3}\right)^{3 \times \frac{2}{3}} \div (2^5)^{\frac{-2}{5}}$ $\quad \left[\frac{8}{27} = \frac{2 \times 2 \times 2}{3 \times 3 \times 3} = \left(\frac{2}{3}\right)^3 \text{ and } 32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5\right]$

$= \left(\frac{2}{3}\right)^2 \div 2^{-2}$ $\quad \left[3 \times \frac{2}{3} = 2 \text{ and } 5 \times \frac{-2}{5} = -2\right]$

$= \frac{2^2}{3^2} \times \frac{1}{2^{-2}}$

$= \frac{4}{9} \times 2^2$ $\quad \left[\frac{1}{2^{-2}} = 2^2\right]$

$= \frac{4 \times 4}{9} = \frac{16}{9} = 1\frac{7}{9}$

(Ans.)

(iii) **Given expression**

$$= -2^4 - 1 \times 2^6 \div 2^2 \qquad [(\sqrt{3})^0 = 1;\ (-2)^6 = 2^6 \text{ and } 4 = 2 \times 2 = 2^2]$$

$$= -2^4 - 2^4 \qquad [2^6 \div 2^2 = 2^{6-2} = 2^4]$$

$$= -16 - 16 = -32$$ **(Ans.)**

Example 7 :

Simplify : $\dfrac{x^{m+n} \times x^{n+l} \times x^{l+m}}{(x^m \times x^n \times x^l)^2}$

Solution :

Given expression $= \dfrac{x^{m+n+n+l+l+m}}{x^{2m} \times x^{2n} \times x^{2l}}$

$= \dfrac{x^{2m+2n+2l}}{x^{2m+2n+2l}} = 1$ **(Ans.)**

Example 8 :

Simplify : $\left(\dfrac{x^a}{x^b}\right)^{a+b} \times \left(\dfrac{x^b}{x^c}\right)^{b+c} \times \left(\dfrac{x^c}{x^a}\right)^{c+a}$

Solution :

Given expression $= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a}$

$= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)}$

$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$

$= x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1$ **(Ans.)**

EXERCISE 2(B)

1. Compute :
   (i) $1^8 \times 3^0 \times 5^3 \times 2^2$
   (ii) $(4^7)^2 \times (4^{-3})^4$
   (iii) $(2^{-9} \div 2^{-11})^3$
   (iv) $\left(\dfrac{2}{3}\right)^{-4} \times \left(\dfrac{27}{8}\right)^{-2}$
   (v) $\left(\dfrac{56}{28}\right)^0 \div \left(\dfrac{2}{5}\right)^3 \times \dfrac{16}{25}$
   (vi) $(12)^{-2} \times 3^3$
   (vii) $(-5)^4 \times (-5)^6 \div (-5)^9$
   (viii) $\left(-\dfrac{1}{3}\right)^4 \div \left(-\dfrac{1}{3}\right)^8 \times \left(-\dfrac{1}{3}\right)^5$
   (ix) $9^0 \times 4^{-1} \div 2^{-4}$ (x) $(625)^{-\frac{3}{4}}$
   (xi) $\left(\dfrac{27}{64}\right)^{-\frac{2}{3}}$ (xii) $\left(\dfrac{1}{32}\right)^{-\frac{2}{5}}$
   (xiii) $(125)^{-\frac{2}{3}} \div (8)^{\frac{2}{3}}$
   (xiv) $(243)^{\frac{2}{5}} \div (32)^{-\frac{2}{5}}$
   (xv) $(-3)^4 - (\sqrt[4]{3})^0 \times (-2)^5 \div (64)^{\frac{2}{3}}$
   (xvi) $(27)^{\frac{2}{3}} \div \left(\dfrac{81}{16}\right)^{-\frac{1}{4}}$
2. Simplify :
   (i) $8^{\frac{4}{3}} + 25^{\frac{3}{2}} - \left(\dfrac{1}{27}\right)^{-\frac{2}{3}}$
   (ii) $[(64)^{-2}]^{-3} \div [\{(-8)^2\}^3]^2$
   (iii) $(2^{-3} - 2^{-4})(2^{-3} + 2^{-4})$
3. Evaluate :
   (i) $(-5)^0$ (ii) $8^0 + 4^0 + 2^0$
   (iii) $(8 + 4 + 2)^0$ (iv) $4x^0$
   (v) $(4x)^0$ (vi) $[(10^3)^0]^5$
   (vii) $(7x^0)^2$
   (viii) $9^0 + 9^{-1} - 9^{-2} + 9^{\frac{1}{2}} - 9^{-\frac{1}{2}}$

4. Simplify :

(i) $\dfrac{a^5 b^2}{a^2 b^{-3}}$ (ii) $15y^8 \div 3y^3$

(iii) $x^{10}y^6 \div x^3y^{-2}$ (iv) $5z^{16} \div 15z^{-11}$

(v) $(36x^2)^{\frac{1}{2}}$ (vi) $(125x^{-3})^{\frac{1}{3}}$

(vii) $(2x^2y^{-3})^{-2}$ (viii) $(27x^{-3}y^6)^{\frac{2}{3}}$

(ix) $(-2x^{2/3}y^{-3/2})^6$

5. Simplify : $(x^{a+b})^{a-b}.(x^{b+c})^{b-c}.(x^{c+a})^{c-a}$

6. Simplify : (i) $\sqrt[5]{x^{20}y^{-10}z^5} \div \dfrac{x^3}{y^3}$

(ii) $\left(\dfrac{256a^{16}}{81b^4}\right)^{\frac{-3}{4}}$

7. Simplify and express as positive indices :

(i) $(a^{-2}b)^{-2}.(ab)^{-3}$ (ii) $(x^ny^{-m})^4 \times (x^3y^{-2})^{-n}$

(iii) $\left(\dfrac{125a^{-3}}{y^6}\right)^{\frac{-1}{3}}$ (iv) $\left(\dfrac{32x^{-5}}{243y^{-5}}\right)^{\frac{-1}{5}}$

(v) $(a^{-2}b)^{\frac{1}{2}} \times (ab^{-3})^{\frac{1}{3}}$

(vi) $(xy)^{m-n} \cdot (yz)^{n-l} \cdot (zx)^{l-m}$

8. Show that :

$\left(\dfrac{x^a}{x^{-b}}\right)^{a-b} \cdot \left(\dfrac{x^b}{x^{-c}}\right)^{b-c} \cdot \left(\dfrac{x^c}{x^{-a}}\right)^{c-a} = 1$

9. Evaluate : $\dfrac{x^{5+n} \times (x^2)^{3n+1}}{x^{7n-2}}$

10. Evaluate : $\dfrac{a^{2n+1} \times a^{(2n+1)(2n-1)}}{a^{n(4n-1)} \times (a^2)^{2n+3}}$

11. Prove that : $(m+n)^{-1}(m^{-1}+n^{-1}) = (mn)^{-1}$

12. Prove that :

(i) $\left(\dfrac{x^a}{x^b}\right)^{\frac{1}{ab}} \left(\dfrac{x^b}{x^c}\right)^{\frac{1}{bc}} \left(\dfrac{x^c}{x^a}\right)^{\frac{1}{ca}} = 1$

(ii) $\dfrac{1}{1+x^{a-b}} + \dfrac{1}{1+x^{b-a}} = 1$

13. Find the value of n, when :

(i) $12^{-5} \times 12^{2n+1} = 12^{13} \div 12^7$

(ii) $\dfrac{a^{2n-3} \times (a^2)^{n+1}}{(a^4)^{-3}} = (a^3)^3 \div (a^6)^{-3}$

14. Simplify :

(i) $\dfrac{a^{2n+3} \cdot a^{(2n+1)(n+2)}}{(a^3)^{2n+1} \cdot a^{n(2n+1)}}$

(ii) $\dfrac{x^{2n+7} \cdot (x^2)^{3n+2}}{x^{4(2n+3)}}$

SQUARES AND SQUARE ROOTS 3

3.1 REVIEW

1. **Square**	If a number is multiplied by itself, the product obtained is called the square of that number. *e.g.* (i) Since, $5 \times 5 = 25$; $\therefore$ 25 is square of 5 and we write $(5)^2 = 25$. (ii) 0·04 is square of 0·2 as $0{\cdot}2 \times 0{\cdot}2 = 0{\cdot}04$ and so on.
2. **Square root**	The **square root** of a given number **x** is the number whose square is **x**. *e.g.* square root of 36 is 6 as square of 6 is 36 *i.e.* $6^2 = 36$. The symbol of square root is radical sign $\sqrt{\ }$. Thus, square root of $64 = \sqrt{64} = 8$; square root of $1{\cdot}44 = \sqrt{1{\cdot}44} = 1{\cdot}2$ and so on. The sign $\sqrt{\ }$ is of the form of letter r, the first letter of the Latin word **radix** meaning **a root**.

1. $4^2 = 16$ is also read as; 4 **raised to the power** 2 is 16.
2. Squares of even numbers are always even.
   e.g. $2^2 = 2 \times 2 = 4$; $6^2 = 6 \times 6 = 36$; $14^2 = 14 \times 14 = 196$ and so on.
3. Squares of odd numbers are always odd.
   e.g. $3^2 = 3 \times 3 = 9$; $7^2 = 49$; $15^2 = 225$ and so on.
4. Whether the number is negative or positive, its square is always positive.
   e.g. $(3)^2 = 3 \times 3 = 9$, **which is a positive number.**
   $(-3)^2 = -3 \times -3 = 9$, **which is also a positive number.**
   Similarly, $(-5)^2 = 25$ and $(5)^2 = 25$, $(-8)^2 = 64$ and $8^2 = 64$.
5. Since, the square of every number is positive, the square root of a positive number can be obtained, but the square root of a negative number is not possible.

3.2 PERFECT SQUARE

A number, whose exact square root can be obtained, is called a **perfect square**.

e.g. 16, 49, 1·21, $\frac{9}{16}$, etc. are perfect squares as $\sqrt{16} = 4$, $\sqrt{49} = 7$, $\sqrt{1{\cdot}21} = 1{\cdot}1$ and so on.

To find out whether a given number is a perfect square or not, express the number as a product of its prime factors. If the number is a perfect square, you would be able to group all the factors in pairs in such a way that both the factors in each pair are equal.

Example 1 :

Is 196 a perfect square ?

Solution :

$196 = 2 \times 2 \times 7 \times 7 = \overline{2 \times 2} \times \overline{7 \times 7}$

$\because$ The prime factors of 196 can be grouped in pairs; **196 is a perfect square. (Ans.)**

Example 2 :

Is 180 a perfect square ?

Solution :

$180 = 2 \times 2 \times 3 \times 3 \times 5 = \overline{2 \times 2} \times \overline{3 \times 3} \times 5$

Since, all the prime factors of 180 cannot be grouped in pairs. [One factor (*i.e.* 5) is left]

∴ **180 is not a perfect square.** **(Ans.)**

3.3 TO FIND THE SQUARE ROOT OF A PERFECT SQUARE NUMBER

(Using Prime Factor Method)

Example 3 :

Find the square root of 484.

Solution :

Square root of 484 $= \sqrt{484}$

Steps :

1. Resolve the number into prime factors : $= \sqrt{2 \times 2 \times 11 \times 11}$
2. Make pairs such that both the factors in each pair are equal : $= \sqrt{(2 \times 2) \times (11 \times 11)}$
3. Take one factor from each pair : $= 2 \times 11$
4. **The product is the square root of the given number** $= 22$ **(Ans.)**

Example 4 :

Find the smallest number by which 980 be multiplied so that the product is a perfect square.

Solution :

$980 = \overline{2 \times 2} \times 5 \times \overline{7 \times 7}$

Since, the prime factor 5 is not in pair.

∴ **The given number should be multiplied by 5.** **(Ans.)**

$980 \times 5 = \overline{2 \times 2} \times \overline{5 \times 5} \times \overline{7 \times 7}, \quad \therefore \sqrt{980 \times 5} = 2 \times 5 \times 7 = 70$

Example 5 :

Find the smallest number by which 3150 be divided, so that the quotient is a perfect square.

Solution :

$3150 = 2 \times \overline{5 \times 5} \times \overline{3 \times 3} \times 7$

Since, the prime factors 2 and 7 cannot be paired.

∴ **The given number should be divided by $2 \times 7 = 14$** **(Ans.)**

$\frac{3150}{14} = \frac{2 \times \overline{5 \times 5} \times \overline{3 \times 3} \times 7}{2 \times 7} = \overline{5 \times 5} \times \overline{3 \times 3}$

Example 6 :

Find the square root of : (i) $2\frac{7}{9}$ (ii) 4·41

Solution :

(i) Square root of $2\frac{7}{9} = \sqrt{2\frac{7}{9}} = \sqrt{\frac{25}{9}} = \frac{5}{3} = 1\frac{2}{3}$ **(Ans.)**

> *Square root of a fraction* $= \dfrac{\textit{Square root of its numerator}}{\textit{Square root of its denominator}}$

(ii) $\sqrt{4\cdot41} = \sqrt{\frac{441}{100}} = \sqrt{\frac{3\times3\times7\times7}{2\times2\times5\times5}} = \frac{3\times7}{2\times5} = \frac{21}{10} = 2\cdot1$ **(Ans.)**

1. Instead of writing the prime factors of the given number in pairs, we can write them in index form and then in order to find the required square root, take half of each index value.

   e.g. $\sqrt{784} = \sqrt{2\times2\times2\times2\times7\times7} = \sqrt{2^4\times7^2} = 2^2\times7^1 = 28$

2. $\sqrt{9} = 3$, but $\sqrt{0\cdot9} \neq 0\cdot3$

   Reason : $(0\cdot3)^2 = 0\cdot3\times0\cdot3 = 0\cdot09 \quad \therefore \quad \sqrt{0\cdot09} = 0\cdot3$

   In the same way, $\sqrt{144} = 12$, but $\sqrt{14\cdot4} \neq 1\cdot2$

   Reason : $(1\cdot2)^2 = 1\cdot2\times1\cdot2 = 1\cdot44 \quad \therefore \quad \sqrt{1\cdot44} = 1\cdot2$

3. Square root of a perfect square even number is always an even number and square root of a perfect square odd number is always an odd number.

   e.g. (i) $\sqrt{4} = 2$, $\sqrt{16} = 4$, $\sqrt{36} = 6$, $\sqrt{64} = 8$, $\sqrt{100} = 10$ and so on.

   (ii) $\sqrt{9} = 3$, $\sqrt{25} = 5$, $\sqrt{49} = 7$, $\sqrt{81} = 9$, $\sqrt{121} = 11$ and so on.

Example 7 :

A man plants his orchard with 5625 trees and arranges them so that there are as many rows as there are trees in each row. How many rows are there ?

Solution :

Let the number of rows be x.

$\therefore$ Number of trees in each row = x

and, total number of trees planted = $x \times x = x^2$

Given : $x^2 = 5625 \Rightarrow x = \sqrt{5625} = \sqrt{\overline{5\times5}\times\overline{5\times5}\times\overline{3\times3}}$

$= 5\times5\times3 = 75$

$\therefore$ **The number of rows = 75** **(Ans.)**

Example 8 :

In a basket there are 50 flowers. A man goes to worship and puts as many flowers in each temple as there are temples in the city. Thus, he needs 8 baskets of flowers. Find the number of temples in the city.

Solution :

Let the number of temples in the city = x

$\therefore$ The number of flowers put in each temple = x

and, the total number of flowers used = $x \times x = x^2$

According to the given statement :

$x^2 = 50\times8 \Rightarrow x = \sqrt{50\times8} = \sqrt{\overline{5\times5}\times\overline{2\times2}\times\overline{2\times2}} = 5\times2\times2 = 20$

$\therefore$ **The number of temples in the city = 20** **(Ans.)**

Example 9 :

Find the smallest perfect square number, which is divisible by 8 and 12.

Solution :

The required smallest perfect square number divisible by 8 and 12 is divisible by L.C.M. of 8 and 12

Since, L.C.M. of 8 and 12 = 24 and $24 = \underline{2 \times 2} \times 2 \times 3$

To make it a perfect square, it must be multiplied by 2 and 3.

$\therefore$ **Required perfect square number** $= 24 \times 2 \times 3 = 144$ (Ans.)

Example 10 :

Find the smallest perfect square number divisible by 24, 30 and 60.

Solution :

$\because$ L.C.M. of 24, 30 and 60 = 120

and $120 = \underline{2 \times 2} \times 2 \times 3 \times 5$ which will be the smallest perfect square on multiplying it with $2 \times 3 \times 5$.

$\therefore$ **Required perfect square number** $= 120 \times 2 \times 3 \times 5 = 3600$ (Ans.)

EXERCISE 3(A)

1. Find the square of :
   (i) 59 (ii) 6·3 (iii) 15
2. By splitting into prime factors, find the square root of :
   (i) 11025 (ii) 396900 (iii) 194481
3. (i) Find the smallest number by which 2592 be multiplied so that the product is a perfect square.
   (ii) Find the smallest number by which 12748 be multiplied so that the product is a perfect square.
4. Find the smallest number by which 10368 be divided, so that the result is a perfect square. Also, find the square root of the resulting number.
5. Find the square root of :
   (i) 0·1764 (ii) $96\frac{1}{25}$ (iii) 0·0169
6. Evaluate :
   (i) $\sqrt{\frac{14\cdot4}{22\cdot5}}$ (ii) $\sqrt{\frac{0\cdot225}{28\cdot9}}$
   (iii) $\sqrt{\frac{25}{32} \times 2\frac{13}{18} \times 0\cdot25}$
   (iv) $\sqrt{1\frac{4}{5} \times 14\frac{21}{44} \times 2\frac{7}{55}}$
7. Evaluate :
   (i) $\sqrt{3^2 \times 6^3 \times 24}$
   (ii) $\sqrt{(0\cdot5)^3 \times 6 \times 3^5}$
   (iii) $\sqrt{\left(5+2\frac{21}{25}\right) \times \frac{0\cdot169}{1\cdot6}}$
   (iv) $\sqrt{5\left(2\frac{3}{4} - \frac{3}{10}\right)}$
   (v) $\sqrt{248 + \sqrt{52 + \sqrt{144}}}$
8. A man, after a tour, finds that he had spent every day as many rupees as the number of days he had been on tour. How long did his tour last, if he had spent in all ₹ 1,296?
9. Out of 745 students, maximum are to be arranged in the school field for a P.T. display such that the number of rows is equal to the number of columns. Find the number of rows if 16 students were left out after the arrangement.
10. 13 and 31 is a strange pair of numbers such that their squares 169 and 961 are also mirror images of each other. Find two more such pairs.
11. Find the smallest perfect square divisible by 3, 4, 5 and 6.

L.C.M. of 3, 4, 5 and 6 = 60.
Also, $60 = 2 \times 2 \times 5 \times 3$ in which 5 and 3 are not in pairs.
So, 60 should be multiplied by 5×3 to get a perfect square number.
∴ **The required least square number**
$= 60 \times 5 \times 3 =$ **900** **Ans.**

12. If $\sqrt{784} = 28$, find the value of :
(i) $\sqrt{7{\cdot}84} + \sqrt{78400}$
(ii) $\sqrt{0{\cdot}0784} + \sqrt{0{\cdot}000784}$

6.4 TO FIND THE SQUARE ROOT OF A PERFECT SQUARE NUMBER
(Using Division Method)

Example 11 :

Find the square root of 276676

Solution :

Steps :

1. Group the digits in pairs starting from right to left, thus $276676 = \overline{27}\ \overline{66}\ \overline{76}$. Take the first pair (*i.e.* 27) and find the largest whole number which when multiplied by itself gives 27 or just less than 27. Such a number is 5.
   Write 5 in the quotient and also in the divisor.
2. Subtract $5 \times 5 = 25$ from 27 (the first pair).
   The remainder is 2.
3. Bring down the second pair of digits (*i.e.* 66).
   Double the quotient (*i.e.* 5) and write the result ($5 \times 2 = 10$) on the left of 266.
   Put the largest possible digit on the right of 10, such that the product of this largest digit with the new number obtained does not exceed 266. Such a digit is 2 as $102 \times 2 = 204$.
   Write 2 in the quotient also.
4. Subtract $102 \times 2 = 204$ from 266. The remainder is 62.
   Bring down the next pair of digits (*i.e.* 76) and proceed as in (3).

	5 2 6	
5	$\overline{27}\ \overline{66}\ \overline{76}$	(Step 1)
	25	(Step 2)
102	2 66	(Step 3)
	2 04	
1046	62 76	(Step 4)
	62 76	
	×	

∴ $\sqrt{276676} = 526$ **(Ans.)**

Example 12 :

Using the division method find the square root of : (i) 4489 (ii) 46656

Solution :

(i)

	6 7
6	$\overline{44}\ \overline{89}$
	36
127	8 89
	8 89
	×

∴ $\sqrt{4489} = 67$ **(Ans.)**

(ii)

	2 1 6
2	$4\ \overline{66}\ \overline{56}$
	4
41	× 66
	41
426	25 56
	25 56
	×

∴ $\sqrt{46656} = 216$ **(Ans.)**

Example 13 :

Using the division method find the square root of : (i) 605·16 (ii) 0·000729

Solution :

In the mixed decimal numbers, starting from the decimal point, group the integral part from right to left and decimal part from left to right.

Thus, $605{\cdot}16 = 6\,\overline{05}\cdot\overline{16}$ and $0{\cdot}000729 = 0\cdot\overline{00}\,\overline{07}\,\overline{29}$

Now, proceed exactly in the same way as explained above. Just remember to put a decimal in the quotient as the decimal point in the dividend is crossed.

(i)

	24·6
2	$6\,\overline{05}\cdot\overline{16}$
	4
44	2 05
	1 76
486	29 16
	29 16
	×

$\therefore\ \sqrt{605{\cdot}16} = 24{\cdot}6$ **(Ans.)**

(ii)

	0·027
2	$0\cdot\overline{00}\,\overline{07}\,\overline{29}$
	4
47	3 29
	3 29
	×

$\therefore\ \sqrt{0{\cdot}000729} = 0{\cdot}027$ **(Ans.)**

Examine the following results :

$\because\ (0{\cdot}7)^2 = 0{\cdot}49 \qquad \therefore\ \sqrt{0{\cdot}49} = 0{\cdot}7$

$\because\ (0{\cdot}03)^2 = 0{\cdot}0009 \qquad \therefore\ \sqrt{0{\cdot}0009} = 0{\cdot}03$

$\because\ (3{\cdot}2)^2 = 10{\cdot}24 \qquad \therefore\ \sqrt{10{\cdot}24} = 3{\cdot}2$

$\because\ (0{\cdot}015)^2 = 0{\cdot}000225 \qquad \therefore\ \sqrt{0{\cdot}000225} = 0{\cdot}015$

It is clear from these results that the square of any decimal number contains even number of decimal places and that the number of decimal places in the square is double the number of decimal places in the square root. Hence, a decimal number (or a mixed decimal number) can be a perfect square only when it has an even number of digits in its decimal part.

3.5 TO FIND THE SQUARE ROOT OF A NUMBER WHICH IS NOT A PERFECT SQUARE *(Using Division Method)*

Example 14 :

Find the square root of 24·729 correct to two places of decimal.

Solution :

When the square root is required correct to two places of decimal, we shall find the square root up to three places of decimal and then round it off upto two places of decimal.

Similarly, if the square root is required correct to three places of decimal, find the square root up to four places and then round it off upto three places and so on.

In order to find square root upto three places of decimal, we must have three pairs of digits after decimal.

For this purpose, $24{\cdot}729 = \overline{24}\cdot\overline{72}\,\overline{90}\,\overline{00}$

(Addition of any number of zeroes on the right of a decimal fraction does not change its value)

```
        4 ·9 7 2
      ___________
 4   | 24·72 90 00
     | 16
     |___________
 89  |  8 72
     |  8 01
     |___________
 987 |    71 90
     |    69 09
     |___________
9942 |     281 00
     |     198 84
     |___________
     |      82 16
```

∴ $\sqrt{24.729}$ = 4·972 upto three places of decimal

= **4·97 correct to two places of decimal** **(Ans.)**

Example 15 :

Find the square root of :

(i) 3, correct to three places of decimal. (ii) 0·07688, correct to two places of decimal.

Solution :

(i) $3 = 3.\overline{00}\,\overline{00}\,\overline{00}\,\overline{00}$

```
         1 ·7 3 2 0
       ______________
    1 | 3· 00 00 00 00
      | 1
      |______________
   27 | 2 00
      | 1 89
      |______________
  343 |   11 00
      |   10 29
      |______________
 3462 |      71 00
      |      69 24
      |______________
34640 |       17600
```

∴ $\sqrt{3}$ = 1·7320

= **1·732** **(Ans.)**

(ii) $0.07688 = 0.\overline{07}\,\overline{68}\,\overline{80}$

```
        0 ·2 7 7
      ____________
  2  | 0· 07 68 80
     |     4
     |____________
 47  |     3 68
     |     3 29
     |____________
 547 |       39 80
     |       38 29
     |____________
     |         151
```

∴ $\sqrt{0.07688}$ = 0·277

= **0·28** **(Ans.)**

Example 16 :

Find the least number that must be subtracted from 2433 so that the remainder is a perfect square.

Solution :

```
      49
    _______
 4 | 24 33
   | 16
   |_______
89 |  8 33
   |  8 01
   |_______
   |    32
```

Clearly, if 32 is subtracted from 2433, the remainder will be a perfect square.

(Ans.)

Since, $2433 - 32 = 2401$
and, $\sqrt{2401} = 49$

Example 17 :

Find the least number which must be added to 18,265 to obtain a perfect square.

Solution :

	135
1	$1\ \overline{82}\ \overline{65}$
	1
23	82
	69
265	13 65
	13 25
	40

Clearly, 18265 is greater than 135^2.

$\therefore$ On adding the required number to 18265, we shall be getting 136^2 *i.e.* 18496

Hence, the required number $= 18496 - 18265$
$= 231$ **(Ans.)**

EXERCISE 3(B)

1. Find the square root of :
   (i) 4761 (ii) 7744 (iii) 15129
   (iv) 0·2916 (v) 0·001225
   (vi) 0·023104 (vii) 27·3529
2. Find the square root of :
   (i) 4·2025 (ii) 531·7636 (iii) 0·007225
3. Find the square root of :
   (i) 245 correct to two places of decimal.
   (ii) 496 correct to three places of decimal.
   (iii) 82·6 correct to two places of decimal.
   (iv) 0·065 correct to three places of decimal.
   (v) 5·2005 correct to two places of decimal.
   (vi) 0·602 correct to two places of decimal.
4. Find the square root of each of the following correct to two decimal places :
   (i) $3\frac{4}{5}$ (ii) $6\frac{7}{8}$

   (i) $3\frac{4}{5} = 3{\cdot}8$ (ii) $6\frac{7}{8} = 6{\cdot}875$
5. For each of the following, find the least number that must be subtracted so that the resulting number is a perfect square.
   (i) 796 (ii) 1886 (iii) 23497
6. For each of the following, find the least number that must be added so that the resulting number is a perfect square.
   (i) 511 (ii) 7172 (iii) 55078
7. Find the square root of 7 correct to two decimal places; then use it to find the value of $\sqrt{\frac{4+\sqrt{7}}{4-\sqrt{7}}}$ correct to three significant digits.

   $$\sqrt{\frac{4+\sqrt{7}}{4-\sqrt{7}}} = \sqrt{\frac{(4+\sqrt{7})(4+\sqrt{7})}{(4-\sqrt{7})(4+\sqrt{7})}}$$
   $$= \sqrt{\frac{(4+\sqrt{7})^2}{16-7}} = \frac{4+\sqrt{7}}{3}$$
8. Find the value of $\sqrt{5}$ correct to 2 decimal places; then use it to find the square root of $\frac{3-\sqrt{5}}{3+\sqrt{5}}$ correct to 2 significant digits.
9. Find the square root of :
   (i) $\frac{1764}{2809}$ (ii) $\frac{507}{4107}$
   (iii) $\sqrt{108 \times 2028}$ (iv) $0{\cdot}01 + \sqrt{0{\cdot}0064}$
10. Find the square root of 7·832 corret to :
    (i) 2 decimal places
    (ii) 2 significant digits.

11. Find the least number which must be subtracted from 1205 so that the resulting number is a perfect square.

By division method, find the square root of 1205

$\because \sqrt{1205} = 34.713 \ldots\ldots$

$\therefore$ **Required number to be subtracted**

$= 1205 - 34^2 = 49$ **(Ans.)**

12. Find the least number which must be added to 1205 so that the resulting number is a perfect square

As done above, $\sqrt{1205} = 34.713 \ldots\ldots$

$\therefore$ **Required number to be added**

$= 35^2 - 1205 =$ **20** **(Ans.)**

13. Find the least number which must be subtracted from 2037 so that the resulting number is a perfect square.

14. Find the least number which must be added to 5483 so that the resulting number is a perfect square.

3.6 PROPERTIES OF SQUARE NUMBERS

1st Property :

The ending digit (*i.e.* the digit at unit's place) of the square of a number is 0, 1, 4, 5, 6 or 9.

For example :

(i) $11^2 = 121$ (ii) $22^2 = 484$ (iii) $53^2 = 2809$ (iv) $30^2 = 900$

(v) $4^2 = 16$ (vi) $25^2 = 625$ (vii) $46^2 = 2116$ (viii) $37^2 = 1369$

(ix) $68^2 = 4624$ (x) $19^2 = 361$

2nd Property :

A number having 2, 3, 7 or 8 at its unit's place is never a perfect square.

For example :

None of the following numbers is a perfect square.

(i) 12, 22, 32, 42, ... (ii) 13, 23, 33, 43, ...

(iii) 17, 27, 37, 47, ... (iv) 18, 28, 38, 48, ...

3rd Property :

If a number has 1 or 9 at its unit's place, then square of this number always has 1(one) at its unit place :

For example :

(i) Square of 1 = 1 (ii) $11^2 = 121$ (iii) $31^2 = 961$

(iv) $9^2 = 81$ (v) $29^2 = 841$ (vi) $49^2 = 2401$ and so on.

4th Property :

If the digit at the unit's place of a number is 4 or 6, then its square will always have 6 at its unit's place.

For example :

(i) $4^2 = 16$ (ii) $6^2 = 36$ (iii) $24^2 = 576$

(iv) $36^2 = 1296$ (v) $84^2 = 7056$ (vi) $96^2 = 9216$ and so on.

5th Property :

If a number ends with n zeroes; its square ends with $2n$ zeroes.

For example :

(i) Square of 30 = 900

(ii) Square of 300 = 90000 and so on.

> The number of zeroes at the end of a square number is always even. That is a number ending in an odd number of zeroes can never be a perfect square.
>
> Thus, each of 40, 360, 49000, 2500000, etc. can not be a perfect square.

6th Property :

A perfect square number leaves remainder 0 or 1 on dividing it by 3.

For example :

(i) 9 is a perfect square number and on dividing it by 3, the remainder is 0.

(ii) 16 is a perfect square number and on dividing it by 3, the remainder is 1.

> When each of perfect square numbers 1, 4, 9, 16, 25, 36, ... is divided by 3, the remainder is either 0 or 1.

7th Property :

For any natural number n,

$$(n + 1)^2 - n^2 = (n + 1) + n$$

For example :

(i) $8^2 - 7^2 = 8 + 7 = 15$

(ii) $15^2 - 14^2 = 15 + 14 = 29$

(iii) $35^5 - 34^2 = 35 + 34 = 69$ and so on.

> 1. The sum of first n odd natural numbers = n^2,
>
> $\Rightarrow$ (i) **1 + 3** = sum of first 2 odd natural numbers = 2^2 = **4**
>
> (ii) **1 + 3 + 5 + 7 + 9** = sum of first 5 odd natural numbers = 5^2 = **25**
>
> (iii) **1 + 3 + 5 + 7 + 9 + ... + 19**
>
> = sum of first 10 odd natural numbers = 10^2 = **100**
>
> 2. For any three natural numbers p, q and r,
>
> if $p^2 + q^2 = r^2$ or $p^2 + r^2 = q^2$ or $q^2 + r^2 = p^2$; the numbers p, q and r are known as **Pythagorean triplets**.
>
> *For example :*
>
> (i) Natural numbers 3, 4 and 5 are Pythagorean triplets as $3^2 + 4^2 = 5^2$.
>
> (ii) $5^2 + 12^2 = 13^2 \Rightarrow$ 5, 12 and 13 are Pythagorean triplets.

EXERCISE 3(C)

1. Seeing the value of the digit at unit's place, state which of the following can be square of a number ?
   (i) 3051 (ii) 2332 (iii) 5684
   (iv) 6908 (v) 50699
2. Squares of which of the following numbers will have 1(one) at their unit's place ?
   (i) 57 (ii) 81 (iii) 139
   (iv) 73 (v) 64
3. Which of the following numbers will not have 1(one) at their unit's place ?
   (i) 32^2 (ii) 57^2 (iii) 69^2
   (iv) 321^2 (v) 265^2
4. Squares of which of the following numbers will not have 6 at their unit's place ?
   (i) 35 (ii) 23 (iii) 64
   (iv) 76 (v) 98
5. Which of the following numbers will have 6 at their unit's place :
   (i) 26^2 (ii) 49^2 (iii) 34^2
   (iv) 43^2 (v) 244^2
6. If a number ends with 3 zeroes, how many zeroes will its square have ?
7. If the square of a number ends with 10 zeroes, how many zeroes will the number have ?
8. Is it possible for the square of a number to end with 5 zeroes ? Give reason.
9. Give reason to show that none of the numbers, given below, is a perfect square.
   (i) 2162 (ii) 6843
   (iii) 9637 (iv) 6598
10. State, whether the square of the following numbers is even or odd ?
    (i) 23 (ii) 54
    (iii) 76 (iv) 75
11. Give reason to show that none of the numbers 640, 81000 and 3600000 is a perfect square.
12. Evaluate :
    (i) $37^2 - 36^2$ (ii) $85^2 - 84^2$
    (iii) $101^2 - 100^2$
13. Without doing the actual addition, find the sum of :
    (i) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23$
    (ii) $1 + 3 + 5 + 7 + 9 + \ldots\ldots\ldots\ldots + 39 + 41$
    (iii) $1 + 3 + 5 + 7 + 9 + \ldots\ldots\ldots\ldots + 51 + 53$
14. Write three sets of Pythagorean triplets such that each set has numbers less than 30

CUBES AND CUBE-ROOTS 4

4.1 INTRODUCTION

For any number m, $m \times m \times m$ is called **cube of m** or **m cube** and is written as m^3.

Thus, $m \times m \times m$ = cube of m

$= m^3 = m$ raised to the power 3, etc.

For example :

(i) Cube of 5 $= 5^3$

$= 5 \times 5 \times 5 = 125$

(ii) Cube of 8 $= 8^3$

$= 8 \times 8 \times 8 = 512$

(iii) Cube of –4 $= -4 \times -4 \times -4$

$= -(4 \times 4 \times 4) = -64$

Cube of a positive number is always positive and cube of a negative number is negative.

4.2 PERFECT CUBE

The cube of a number is called a **perfect cube**.

For example :

(i) $6^3 = 6 \times 6 \times 6 = 216 \Rightarrow$ **216 is a perfect cube.**

(ii) $15^3 = 15 \times 15 \times 15 = 3375 \Rightarrow$ **3375 is a perfect cube.**

Conversely, a given number is a perfect cube, if it can be expressed as the product of triplets of equal factors.

For example :

Consider the number 216, then

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= (2 \times 3) \times (2 \times 3) \times (2 \times 3)$

$= 6 \times 6 \times 6 = (6)^3$

= Product of triplets of equal factors.

$\therefore$ 216 is a perfect cube.

2	216
2	108
2	54
3	27
3	9
	3

Example 1 :

(i) Is 297 a perfect cube ?

(ii) Is 2744 a perfect cube ?

Solution :

(i) $\therefore$ $297 = 3 \times 3 \times 3 \times 11$

$= (3 \times 3 \times 3) \times 11$

Since, triplet of number 11 is not formed,

$\therefore$ 297 is not a perfect cube. Ans.

3	297
3	99
3	33
	11

(ii) $\because$ **2744** $= 2 \times 2 \times 2 \times 7 \times 7 \times 7$

$= (2 \times 7) \times (2 \times 7) \times (2 \times 7)$

$= 14 \times 14 \times 14$

$= (14)^3$

$\therefore$ **2744 is a perfect cube.** **Ans.**

2	2744
2	1372
2	686
7	343
7	49
	7

Example 2 :

What is the smallest number by which 3087 may be multiplied, so that the product is a perfect cube?

Solution :

On finding the prime factors of 3087,

we get : **3087** $= 3 \times 3 \times 7 \times 7 \times 7$

Clearly, **3087 must be multiplied by 3** **Ans.**

3	3087
3	1029
7	343
7	49
	7

3087 × 3 $= (3 \times 3 \times 7 \times 7 \times 7) \times 3$

$= 3 \times 3 \times 3 \times 7 \times 7 \times 7$

$= (3 \times 7) \times (3 \times 7) \times (3 \times 7) = 21 \times 21 \times 21 = (21)^3$

Example 3 :

What is the least number by which 6750 may be divided so that the quotient is a perfect cube ?

Solution :

On finding the prime factors of 6750,

we get : **6750** $= 2 \times 5 \times 5 \times 5 \times 3 \times 3 \times 3$

$= 2 \times (5 \times 5 \times 5) \times (3 \times 3 \times 3)$

Clearly, **6750 must be divided by 2** **Ans.**

2	6750
5	3375
5	675
5	135
3	27
3	9
	3

$\frac{6750}{2} = \frac{2 \times 5 \times 5 \times 5 \times 3 \times 3 \times 3}{2}$

$= 5 \times 5 \times 5 \times 3 \times 3 \times 3$

$= (5 \times 3) \times (5 \times 3) \times (5 \times 3) = 15 \times 15 \times 15 = (15)^3$

1. Cubes of odd natural numbers are odd, as : $1^3 = 1$, $3^3 = 27$, $5^3 = 125$, etc.
2. Cubes of even natural numbers are even, as : $2^3 = 8$, $4^3 = 64$, $6^3 = 216$, etc.

EXERCISE 4(A)

1. Find the cube of :
   (i) 7 (ii) 11 (iii) 16
   (iv) 23 (v) 31 (vi) 42
   (vii) 54

2. Find which of the following are perfect cubes ?
   (i) 243 (ii) 588 (iii) 1331
   (iv) 24000 (v) 1728 (vi) 1938

3. Find the cubes of :
   (i) 2·1 (ii) 0·4 (iii) 1·6
   (iv) 2·5 (v) 0·12 (vi) 0·02
   (vii) 0·8

> (v) Cube of 0·12 $= (0{\cdot}12)^3$
> $= 0{\cdot}12 \times 0{\cdot}12 \times 0{\cdot}12$
> $= \mathbf{0{\cdot}001728}$

4. Find the cubes of :
   (i) $\frac{3}{7}$ (ii) $\frac{8}{9}$ (iii) $\frac{10}{13}$
   (iv) $1\frac{2}{7}$ (v) $2\frac{1}{2}$

> (v) Cube of $2\frac{1}{2} = \left(2\frac{1}{2}\right)^3$
> $= \left(\frac{5}{2}\right)^3 = \frac{5 \times 5 \times 5}{2 \times 2 \times 2}$
> $= \frac{125}{8} = 15\frac{5}{8}$

5. Find the cubes of :
   (i) −3 (ii) −7 (iii) −12
   (iv) −18 (v) −25 (vi) −30
   (vii) −50
6. Which of the following are cubes of :
   (i) an even number
   (ii) an odd number.
   216, 729, 3375, 8000, 125, 343, 4096 and 9261.
7. Find the least number by which 1323 must be multiplied so that the product is a perfect cube.
8. Find the smallest number by which 8768 must be divided so that the quotient is a perfect cube.
9. Find the smallest number by which 27783 be multiplied to get a perfect square number.
10. With what least number must 8640 be divided so that the quotient is a perfect cube ?
11. Which is the smallest number that must be multiplied to 77175 to make it a perfect cube?

4.3 CUBE-ROOTS

The cube-root of a given number is the number whose cube is the given number.

⇒ if cube-root of number x is y, then cube of y is x.

⇒ if $\sqrt[3]{x} = y$, then $y^3 = x$.

For example :

(i) cube of 3 = 27 ⇒ cube-root of 27 = 3 *i.e.* $\sqrt[3]{27} = 3$

(ii) cube of 7 = 343 ⇒ cube-root of 343 = 7 *i.e.* $\sqrt[3]{343} = 7$ and so on.

In the same way :

(i) $125 = 5^3 \Rightarrow \sqrt[3]{125} = 5$

(ii) $512 = 8^3 \Rightarrow \sqrt[3]{512} = 8$ and so on.

4.4 CUBE-ROOT BY FACTORISATION

Steps :

1. Split the given number into its primes.
2. Form groups in triplets of the identical primes.
3. Take one prime number from each triplet.
4. Multiply all the prime numbers obtained in step 3 to get the required cube-root.

Example 4 :

Find the cube-root of 216.

2	216
2	108
2	54
3	27
3	9
	3

Solution :

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= (2 \times 2 \times 2) \times (3 \times 3 \times 3)$

$\Rightarrow \quad \sqrt[3]{216} = 2 \times 3 = 6$ **Ans.**

Example 5 :

Find the cube root 1728.

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

Solution :

$1728 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3)$

$\Rightarrow$ **Cube-root of 1728** $= 2 \times 2 \times 3 = 12$ **Ans.**

Example 6 :

Find the smallest number by which 210125 must be multiplied so that the product is a perfect cube.

5	210125
5	42025
5	8405
41	1681
	41

Solution :

$210125 = 5 \times 5 \times 5 \times 41 \times 41$

$= (5 \times 5 \times 5) \times 41 \times 41$

Clearly, **the required smallest number = 41** **Ans.**

1. Cube-root of a negative perfect cube :

If m is any positive integer, then $-m$ is a negative integer.

Since, $(-m)^3 = -m \times -m \times -m$

$= -m^3$

$\therefore \quad \sqrt[3]{-m^3} = -\sqrt[3]{m^3} = -m$

Cube-root of $(-m^3)$ = – (cube-root of m^3).

For example :

1. Cube-root of -8 = – (cube-root of 8)

$= -2$

$\Rightarrow \quad \sqrt[3]{-8} = -2$

2. Cube-root of -1000 = – (cube-root of 1000)

$= -10$

i.e. $\quad \sqrt[3]{-1000} = -10$

3. Cube-root of − 1 = − (cube root of 1)

$= -1$

i.e. $\sqrt[3]{-1} = -1$ and so on.

2. Cube-root of product of numbers :

(a) $\sqrt[3]{xy} = \sqrt[3]{x} \times \sqrt[3]{y}$

(b) $\sqrt[3]{xyz} = \sqrt[3]{x} \times \sqrt[3]{y} \times \sqrt[3]{z}$ and so on.

For example :

1. $\sqrt[3]{8 \times 125} = \sqrt[3]{8} \times \sqrt[3]{125}$

$= 2 \times 5 = 10$

2. $\sqrt[3]{500 \times 54} = \sqrt[3]{500 \times 2 \times 27}$

$= \sqrt[3]{1000 \times 27}$

$= \sqrt[3]{1000} \times \sqrt[3]{27} = 10 \times 3 = 30$

3. $\sqrt[3]{-16 \times 32} = \sqrt[3]{-8 \times 2 \times 32}$

$= \sqrt[3]{-8 \times 64}$

$= \sqrt[3]{-8} \times \sqrt[3]{64} = -2 \times 4 = -8$

4. $\sqrt[3]{216 \times -343} = \sqrt[3]{216} \times \sqrt[3]{-343}$

$= 6 \times -7 = -42$

3. Cube-root of fractional numbers :

(a) $\sqrt[3]{\frac{x}{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}}$

(b) $\sqrt[3]{\frac{xy}{z}} = \frac{\sqrt[3]{x} \times \sqrt[3]{y}}{\sqrt[3]{z}}$

(c) $\sqrt[3]{\frac{x}{yz}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y} \times \sqrt[3]{z}}$ and so on.

For example :

1. $\sqrt[3]{\frac{125}{216}} = \frac{\sqrt[3]{125}}{\sqrt[3]{216}} = \frac{\sqrt[3]{5 \times 5 \times 5}}{\sqrt[3]{6 \times 6 \times 6}} = \frac{5}{6}$

2. $\sqrt[3]{\frac{-8}{27}} = \frac{\sqrt[3]{(-2) \times (-2) \times (-2)}}{\sqrt[3]{3 \times 3 \times 3}} = \frac{-2}{3}$

3. $\sqrt[3]{\frac{64}{-125}} = \frac{\sqrt[3]{4\times4\times4}}{\sqrt[3]{(-5)\times(-5)\times(-5)}} = \frac{4}{-5} = -\frac{4}{5}$

4. Cube-root of a decimal number :

Convert the given decimal number into a fractional number and then find its cube-root.

For example :

1. $\sqrt[3]{0{\cdot}027} = \sqrt[3]{\frac{0027}{1000}} = \sqrt[3]{\frac{27}{1000}} = \frac{3}{10} = 0{\cdot}3$

2. $\sqrt[3]{0{\cdot}008} = \sqrt[3]{\frac{0008}{1000}} = \sqrt[3]{\frac{8}{1000}} = \frac{2}{10} = 0{\cdot}2$

3. $\sqrt[3]{0{\cdot}125} = \sqrt[3]{\frac{125}{1000}} = \frac{5}{10} = 0{\cdot}5$

EXERCISE 4(B)

1. Find the cube-roots of :
   (i) 64 (ii) 343
   (iii) 729 (iv) 1728
   (v) 9261 (vi) 4096
   (vii) 8000 (viii) 3375

2. Find the cube-roots of :
   (i) $\frac{27}{64}$ (ii) $\frac{125}{216}$
   (iii) $\frac{343}{512}$ (iv) 64×729
   (v) 64×27 (vi) 729×8000
   (vii) 3375×512

3. Find the cube-roots of :
   (i) −216 (ii) −512 (iii) −1331
   (iv) $-\frac{27}{125}$ (v) $\frac{-64}{343}$ (vi) $-\frac{512}{343}$
   (vii) −2197 (viii) −5832 (ix) −2744000

4. Find the cube-roots of :
   (i) 2·744 (ii) 9·261
   (iii) 0·000027 (iv) −0·512
   (v) −15·625 (vi) -125×1000

5. Find the smallest number by which 26244 should be divided so that the quotient is a perfect cube.

6. What is the least number by which 30375 should be multiplied to get a perfect cube ?

7. Find the cube-roots of :
   (i) $700 \times 2 \times 49 \times 5$ (ii) -216×1728
   (iii) -64×-125 (iv) $-\frac{27}{343}$
   (v) $\frac{729}{-1331}$ (vi) 250·047
   (vii) −175616

PLAYING WITH NUMBERS 5

5.1 INTRODUCTION

In the previous classes, we have learnt about :

1. Natural numbers
2. Whole numbers
3. Integers
4. Rational numbers
5. Factors
6. Multiples
7. Relation between factors and multiples, etc.

5.2 GENERALIZED FORM OF NUMBERS

A **number** is said to be **in generalized** form, if it is expressed as the sum of the product of its digit with their place values.

For example :

(i) 68 = digit 6 × its place value + digit 8 × its place value
$= 6 \times 10 + 8 \times 1$

(ii) 257 = digit 2 × its place value + digit 5 × its place value + digit 7 × its place value
$= 2 \times 100 + 5 \times 10 + 7 \times 1$

(iii) $3479 = 3 \times 1000 + 4 \times 100 + 7 \times 10 + 9 \times 1$

5.3 TWO-DIGIT NUMBERS

Let a two-digit number has ***a*** at its ten's place and ***b*** at its unit's place then;
the number = $10a + b$

Remember : In a two digit number $\mathbf{10a + b}$; the ten's digit number ***a*** is any whole number from 1 to 9 and the unit digit number ***b*** is any whole number from 0 to 9.

Thus,

(i) $75 = 10 \times 7 + 5$
(ii) $68 = 10 \times 6 + 8$
(iii) $50 = 10 \times 5 + 0$ and so on.

In general, any two digit number *ab* made of digits a and b can be written as :
$ab = 10 \times a + b = 10a + b$ [Here, $ab \neq a \times b$]
and, $ba = 10 \times b + a = 10b + a$

5.4 THREE-DIGIT NUMBERS

Let a three-digit number has ***a*** at its hundred's place, ***b*** at its ten's place and ***c*** at unit's place.

The number = $100a + 10b + c$

Remember : In a three digit number $\mathbf{100a + 10b + c}$, the digit ***a*** at hundred's place is any whole number from 1 to 9, the digit ***b*** at ten's place is any whole number from 0 and 9 and the digit ***c*** at unit's place is any whole number from 0 to 9.

Thus,

(i) $428 = 100 \times 4 + 10 \times 2 + 8$ (ii) $300 = 100 \times 3 + 10 \times 0 + 0$

(iii) $579 = 100 \times 5 + 10 \times 7 + 9$

In general a three-digit number *abc* made up of digits *a*, *b* and *c* is written as :

$$abc = 100 \times a + 10 \times b + c$$
$$= 100a + 10b + c,$$
$$bca = 100b + 10c + a,$$
$$cab = 100c + 10a + b \quad \text{and so on.}$$

5.5 SOME INTERESTING PROPERTIES

Property 1 :

Consider a two-digit number $ab = 10a + b$ and the number obtained on reversing its digits $ba = 10b + a$. Then.

$$ab + ba = (10a + b) + (10b + a)$$
$$= 11a + 11b = 11(a + b)$$

$$\Rightarrow \quad a + b = \frac{ab + ba}{11} \text{ and } 11 = \frac{ab + ba}{a + b}$$

$\Rightarrow$ The sum of a two digit number *ab* and the number *ba* obtained on reversing its digits is completely divisible by (i) 11 and (ii) the sum of its digits *i.e.* $a + b$.

For example :

1. **Consider the two digit number 35**

   The number obtained on reversing its digits = 53

   Sum of these two numbers = 35 + 53 = 88

   Now, $\frac{88}{11} = 8 \Rightarrow$ the sum of 35 and 53 is divisible by 11.

   Also, $\frac{88}{3+5} = \frac{88}{8} = 11 \Rightarrow$ the sum of 35 and 53 is divisible by the sum of the digits 3 and 5.

2. **Consider the two digit number 87**

   The number obtained on reversing its digits = 78

   Sum of these two digit numbers = 87 + 78 = 165

   Check whether 165 is divisible by 11 or not.

   Also, check whether 165 is divisible by 8 + 7 = 15 or not.

   Yes, 165 is divisible by 11 as $\frac{165}{11} = 15$

   Also, 165 is divisible by sum of the digits 8 and 7

   i.e. 165 is divisible by 8 + 7 = 15 as $\frac{165}{15} = 11$.

When $ab + ba$ is divided by 11, the quotient = $a + b$
and, when $ab + ba$ is divided by $a + b$, the quotient = 11

Example 1 :

Is the sum of two digit numbers 62 and 26 divisible by 8 and 11 ? Explain.

Solution :

$\because$ Out of 62 and 26, one number can be obtained by interchanging the digits of the other

Also, $6 + 2 = 8$; therefore when the sum of two given numbers (62 and 26) is divided by 8, the quotient will be 11 and when divided by 11, the quotient will be 8.

$\Rightarrow$ Sum of 62 and 26 is divisible by 8 and 11 both.

Property 2 :

Consider a two-digit number $ab = 10a + b$ and the number obtained on reversing its digits $ba = 10b + a$. Then

1. $ab - ba = (10a + b) - (10b + a)$
$= 10a + b - 10b - a$
$= 9a - 9b$
$ab - ba = 9(a - b)$
$\Rightarrow \dfrac{ab - ba}{9} = a - b$ and $\dfrac{ab - ba}{a - b} = 9$

2. $ba - ab = (10b + a) - (10a + b)$
$= 10b + a - 10a - b$
$= 9b - 9a$
$\Rightarrow ba - ab = 9(b - a)$
$\Rightarrow \dfrac{ba - ab}{9} = b - a$ and $\dfrac{ba - ab}{b - a} = 9$

$\Rightarrow$ The difference between a two digit number *ab* and the two digit number *ba*, obtained on reversing the digits, is completely divisible by :

(i) 9 and (ii) the difference between its digits

- If $a > b$; $ab > ba$ $\Rightarrow$ (i) on dividing $ab - ba$ by 9, quotient $= a - b$.
  (ii) on dividing $ab - ba$ by $a - b$, quotient $= 9$.
- If $b > a$; $ba > ab$ $\Rightarrow$ (i) on dividing $ba - ab$ by 9, quotient $= b - a$.
  (ii) on dividing $ba - ab$ by $b - a$, quotient $= 9$.

For example :

1. **Consider the two digit number 73**

   The number obtained on reversing its digits = 37

   The difference between these two numbers = 73 − 37 = 36

   Now, $\dfrac{36}{9} = 4 \Rightarrow$ the difference between 73 and 37 is divisible by 9.

   Also, $\dfrac{36}{7-3} = \dfrac{36}{4} = 9 \Rightarrow$ the difference between 73 and 37 is divisible by the difference between its digits 7 and 3.

2. **Consider the two digit number 38**

   The number obtained on reversing its digits = 83

   The difference between 38 and 83 = 83 − 38 = 45

   Now, $\dfrac{45}{9} = 5 \Rightarrow$ the difference between 38 and 83 is divisible by 9.

   Also, $\dfrac{45}{8-3} = \dfrac{45}{5} = 9 \Rightarrow$ the difference between 38 and 83 is divisible by the difference between its digits 8 and 3.

Example 2 :

Find the quotient when 83 – 38 is divided by (i) 9 (ii) 5.

Solution :

(i) $\because$ When $ab - ba$ is divided by 9, quotient is $a - b$

$\therefore$ When 83 – 38 is divided by 9, the **quotient is 8 – 3 = 5** **(Ans.)**

(ii) $\because$ When $ab - ba$ is divided by $a - b$, the quotient is 9

$\Rightarrow$ When 83 – 38 is divided by 8 – 3 = 5, the **quotient is 9** **(Ans.)**

Property 3 :

Consider a 3-digit number *abc*. On changing its digits in order as shown ahead (a)*bc* ; (b)*ca* ; *cab*, we get two more 3-digit numbers *bca* and *cab*. Clearly

$abc = 100a + 10b + c$, $\quad bca = 100b + 10c + a \quad$ and $\quad cab = 100c + 10a + b$

Now, $\quad abc + bca + cab = 111a + 111a + 111c$

$\qquad\qquad = 111\,(a + b + c)$

$$\therefore \frac{abc + bca + cab}{111} = (a + b + c) \quad \text{and} \quad \frac{abc + bca + cab}{a + b + c} = 111$$

$\Rightarrow$ The sum of a three digit number and the two numbers obtained by changing its digits in order is completely divisible by (i) 111 and (ii) sum of its digits.

For example : Consider a three digit number 374. The two numbers obtained on changing its digits in order are 743 and 437.

Adding them, we get : 374 + 743 + 437 = 1554 ; sum of the digits *i.e.* 3 + 7 + 4 = 14.

$$\text{We have } \frac{1554}{(3+7+4)} = 111 \quad i.e. \quad \frac{1554}{14} = 111 \quad \text{and} \quad \frac{1554}{111} = 14 = (3 + 7 + 4)$$

$\Rightarrow$ The sum of 374, 743 and 437 is divisible by both 111 and the sum of digits *i.e.* 14.

Example 3 :

Find the quotient when 821 + 218 + 182 is divided by 111. Will the sum also be divisible by 11 ? Explain.

Solution :

$\because$ When $abc + bca + cab$ is divided by 111, the quotient is $a + b + c$.

$\therefore$ When 821 + 218 + 182 is divided by 111, the quotient will be 8 + 2 + 1 = **11**.

$\because$ When $abc + bca + cab$ is divided by $a + b + c$, the quotient is 111.

We have 8 + 2 + 1 = 11. $\therefore$ 821 + 218 + 182 will be divisible by 11.

EXERCISE 5(A)

1. Write the quotient when the sum of 73 and 37 is divided by :
   (i) 11 (ii) 10
2. Write the quotient when the sum of 94 and 49 is divided by :
   (i) 11 (ii) 13
3. Find the quotient when 73 – 37 is divided by : (i) 9 (ii) 4
4. If $a = b$, show that $abc = bac$.
5. Find the quotient when 94 – 49 is divided by :
   (i) 9 (ii) 5
6. Show that 527 + 752 + 275 is exactly divisible by 14.
7. if $a > c$; show that $abc - cba = 99(a - c)$.
8. If $c > a$; show that $cba - abc = 99(c - a)$.
9. If $a = c$, show that $cba - abc = 0$.

5.6 LETTERS FOR DIGITS (CRYPTARITHMS)

Cryptarithm is a type of mathematical game consisting of numbers, whose digits are represented by letters and we have to identify the value of each letter.

Example 3 :

Solve the following cryptarithms :

(i)
```
  3 1 A
+ 1 A 3
  5 0 1
```

(ii)
```
  B 9
+ 4 A
  6 5
```

(iii)
```
  8 A 5
+ 9 4 A
1 A 3 3
```

Solution :

(i)
```
  3 1 A
+ 1 A 3
  5 0 1
```

- First, we have to find the value of letter A.
- Clearly, A + 3 is a number whose ones digit is 1.

$\Rightarrow A + 3 = 1, \quad A + 3 = 11, \quad A + 3 = 21$ and so on

$\Rightarrow A = 1 - 3, \quad A = 11 - 3, \quad A = 21 - 3$ and so on

$\Rightarrow A = -2, \quad A = 8, \quad A = 18$ and so on

Since, A is a digit $\therefore A = 8$

- **A = 8** satisfies the addition in tens and hundreds columns.

And so the puzzle can be solved as shown below :

```
  3 1 A         3 1 8
+ 1 A 3   =   + 1 8 3
  5 0 1         5 0 1
```

(Ans.)

(ii)
```
  B 9
+ 4 A
  6 5
```

- Clearly, 9 + A is a number whose ones digit is 5.

$\Rightarrow 9 + A = 5, \quad 9 + A = 15, \quad 9 + A = 25$ and so on

$\Rightarrow A = 5 - 9, \quad A = 15 - 9, \quad A = 25 - 9$ and so on

$\Rightarrow A = -4, \quad A = 6, \quad A = 16$ and so on

```
  1
  B 9
+ 4 6
  6 5
```

Since, A is a digit $\therefore A = 6$

$\because$ $9 + A = 9 + 6 = 15$; 5 will come at ones place and digit 1 is carried over

Now, $1 + B + 4 = 6 \Rightarrow B = 1$

$\therefore$ The puzzle can be solved as shown below :

```
  B 9         1 9
+ 4 A   =   + 4 6
  6 5         6 5
```

(Ans.)

(iii)
```
  8 A 5
+ 9 4 A
1 A 3 3
```

- 5 + A must give a number whose ones digit is 3

$\Rightarrow 5 + A = 3, \quad 5 + A = 13, \quad 5 + A = 23,$ ....

$\Rightarrow A = 3 - 5, \quad A = 13 - 5, \quad A = 23 - 5,$ ....

$\Rightarrow A = -2, \quad A = 8, \quad A = 18,$ ....

$\Rightarrow A = 8$, as A is a digit

Writing 8 in place of each A in the given cryptarithm, we get :

```
  8 A 5         8 8 5
+ 9 4 A   =   + 9 4 8
1 A 3 3       1 8 3 3
```

(Ans.)

Example 4 :

Find A and B in the addition :

$$\begin{array}{r} A \\ +A \\ +A \\ \hline BA \end{array}$$

Solution :

- At the ones column, the sum of three As is a number whose ones digit is A.
- This happens only when A = 0 or A = 5
- A = 0 ⇒ A + A + A = 0 + 0 + 0 = 0, which makes B = 0.
- When A = 5, the puzzle is solved as shown alongside, where A = 5 and B = 1.

$$\begin{array}{r} 5 \\ +5 \\ +5 \\ \hline 15 \end{array}$$

Example 5 :

Find the values of A, B and C

$$\begin{array}{r} 4\ 6\ A \\ -C\ B\ 9 \\ \hline 2\ 7\ 5 \end{array}$$

Solution :

Given : 4 6 A − C B 9 = 2 7 5

⇒ 4 6 A = 2 7 5 + C B 9

$$\therefore \begin{array}{r} 4\ 6\ A \\ -C\ B\ 9 \\ \hline 2\ 7\ 5 \end{array} = \begin{array}{r} 2\ 7\ 5 \\ +C\ B\ 9 \\ \hline 4\ 6\ A \end{array}$$

$$\begin{array}{r} {}^{1} \\ 2\ 7\ 5 \\ +C\ B\ 9 \\ \hline 4\ 6\ 4 \end{array}$$

- **In ones column :** 5 + 9 = 14

  ⇒ A = 4 and 1 is carried over

- **In tens column :** 1 + 7 + B = 6

  ⇒ 8 + B is a number with unit digit 6

  ∴ 8 + B = 6 or 16 or 26 or ....

  ⇒ B = −2 or 8 or 18 or ....

  ⇒ B = 8 as B is a digit,

  ∵ 1 + 7 + B = 1 + 7 + 8 = 16

  ∴ In the result, 6 is at tens digit and 1 is carried over.

$$\begin{array}{r} 2\ 7\ 5 \\ +C\ 8\ 9 \\ \hline 4\ 6\ 4 \end{array}$$

$$\begin{array}{r} {}^{1} \\ 2\ 7\ 5 \\ +C\ 8\ 9 \\ \hline 4\ 6\ 4 \end{array}$$

- **At hundreds place :** 1 + 2 + C = 4 ⇒ C = 1

∴ **A = 4, B = 8** and **C = 1** **(Ans.)**

$$\text{Clearly, } \begin{array}{r} 4\ 6\ A \\ -C\ B\ 9 \\ \hline 2\ 7\ 5 \end{array} = \begin{array}{r} 2\ 7\ 5 \\ +C\ B\ 9 \\ \hline 4\ 6\ A \end{array} = \begin{array}{r} 2\ 7\ 5 \\ +1\ 8\ 9 \\ \hline 4\ 6\ 4 \end{array}$$

Example 6 :

Find the values of A and B

$$\begin{array}{r} B\ A \\ \times\ 6 \\ \hline C\ 8\ 8 \end{array}$$

Solution :

∵ A = 8 as 8 × 6 = 48. Clearly, 4 is carried over thus

$$\begin{array}{r} B\ A \\ \times\ 6 \\ \hline C\ 8\ 8 \end{array} = \begin{array}{r} {}^{4} \\ B\ 8 \\ \times\ 6 \\ \hline C\ 8\ 8 \end{array}$$

Now 6 × B + 4 = C8

⇒ 6 × B + 4 = 10 × C + 8 [∵ For a two digit number *ab*, *ab* = 10*a* + *b*]

⇒ 6 × B = 10 × C + 4

The digits which satisfy this equation are B = 4 and C = 2

∴ **A = 8, B = 4** and **C = 2** **(Ans.)**

EXERCISE 5(B)

1. $$\begin{array}{r} 3\;A \\ +\;2\;5 \\ \hline B\;2 \end{array}$$

2. $$\begin{array}{r} 9\;8 \\ +\;4\;A \\ \hline C\;B\;3 \end{array}$$

3. $$\begin{array}{r} A\;1 \\ +\;1\;B \\ \hline B\;0 \end{array}$$

4. $$\begin{array}{r} 2\;A\;B \\ +\;A\;B\;1 \\ \hline B\;1\;8 \end{array}$$

5. $$\begin{array}{r} 1\;2\;A \\ +\;6\;A\;B \\ \hline A\;0\;9 \end{array}$$

6. $$\begin{array}{r} 1\;A \\ \times\;A \\ \hline 9\;A \end{array}$$

7. $$\begin{array}{r} A\;B \\ \times\;6 \\ \hline B\;B\;B \end{array}$$

8. $$\begin{array}{r} A\;B \\ \times\;3 \\ \hline C\;A\;B \end{array}$$

9. $$\begin{array}{r} A\;B \\ \times\;5 \\ \hline C\;A\;B \end{array}$$

10. $$\begin{array}{r} 8\;A\;5 \\ +\;9\;4\;A \\ \hline 1\;A\;3\;3 \end{array}$$

11. $$\begin{array}{r} 6\;A\;B\;5 \\ +\;D\;5\;8\;C \\ \hline 9\;3\;5\;1 \end{array}$$

5.7 TESTS OF DIVISIBILITY

1. **Divisibility by 10 :**

   A number is divisible by 10, if its unit digit is zero.

For example :

Each of 10, 30, 70, 120, 500, etc. is divisible by 10.

2. **Divisibility by 5 :**

   A number is divisible by 5, if its unit digit is 0 or 5.

For example :

Each of 15, 40, 65, 90, 115, 7620, etc. is divisible by 5.

3. **Divisibility by 2 :**

   A number is divisible by 2, if its unit digit is zero or an even number.

For example :

Each of 20, 24, 36, 50, 78, 112, 230, etc. is divisible by 2.

4. **Divisibility by 9 :**

   A number is divisible by 9, if sum of its digits is divisible by 9.

   (i) **Consider the number 45387**

   Sum of its digits = 4 + 5 + 3 + 8 + 7 = 27

   $\because$ 27 is divisible by 9

   $\Rightarrow$ The number **45387 is divisible by 9.**

   (ii) **Consider the number 3518**

   Sum of its digits = 3 + 5 + 1 + 8 = 17

   $\because$ 17 is not divisible by 9

   $\Rightarrow$ The number **3518 is not divisible by 9.**

   (iii) **Consider the number 43242876**

   Sum of its digits = 4 + 3 + 2 + 4 + 2 + 8 + 7 + 6 = 36

   $\because$ 36 is divisible by 9

   $\Rightarrow$ The number **43242876 is divisible by 9.**

5. **Divisibility by 3 :**

A number is divisible by 3, if sum of its digits is divisible by 3.

(i) **Consider the number 645**

Sum of its digits = 6 + 4 + 5 = 15

∵ 15 is divisible by 3.

⇒ The number **645 is divisible by 3.**

(ii) **Consider the number 364028**

Sum of its digits = 3 + 6 + 4 + 0 + 2 + 8 = 23

∵ 23 is not divisible by 3.

⇒ The number **364028 is not divisible by 3.**

6. **Divisibility by 6 :**

A number is divisible by 6, if it is divisible by 2 and 3 both

Consider the number 540

∵ The unit digit of the number is 0, which is divisible by 2

∵ Sum of its digits = 5 + 4 + 0 = 9, which is divisible by 3

⇒ 540 is divisible by 3

∵ **540** is divisible by 2 and 3 both ⇒ it **is divisible by 6.**

7. **Divisibility by 11 :**

A number is divisible by 11, if the difference between the sum of its digits in even places and the sum of its digit in odd places is either 0 or divisible by 11.

(i) **Consider the number 352**

Counting from the right hand side, the sum of its digits in odd places = 2 + 3 = 5

And, the sum of its digits in even places = 5

The difference between these two sums = 5 – 5 = 0

∴ **352 is divisible by 11**

(ii) **Consider the number 61809**

Counting from the right hand side, the sum of its digits in odd places = 9 + 8 + 6 = 23

And, sum of the digits in even places = 0 + 1 = 1

The difference between these two sums = 23 – 1 = 22

∵ 22 is divisible by 11

∴ **61809 is divisible by 11**

8. **Divisibility by 4 :**

A number is divisible by 4, if the two digit number formed by its ten's digit and unit digit is divisible by 4.

(i) **Consider the number 3516**

∵ 16 is divisble by 4

⇒ **3516 is divisible by 4**

(ii) **Consider the number 628093**

∵ 93 is not divisible by 4

⇒ **628093 is not divisible by 4**

EXERCISE 5(C)

1. Find which of the following numbers are divisible by 2 :
   (i) 192 (ii) 1660
   (iii) 1101 (iv) 2079
2. Find which of the following numbers are divsible by 3 :
   (i) 261 (ii) 777
   (iii) 6657 (iv) 2574
3. Find which of the following numbers are divisible by 4 :
   (i) 360 (ii) 3180
   (iii) 5348 (iv) 7756
4. Find which of the following numbers are divisible by 5 :
   (i) 3250 (ii) 5557
   (iii) 39255 (iv) 8204
5. Find which of the following numbers are divisible by 10 :
   (i) 5100 (ii) 4612
   (iii) 3400 (iv) 8399
6. Find which of the following numbers are divisible by 11 :
   (i) 2563 (ii) 8307
   (iii) 95635

Example 7 :

If $42x$ is divisible by 9, find the value of digit x.

Solution :

$\because$ $42x$ is divisible by 9

$\Rightarrow$ $4 + 2 + x$ is divisible by 9

$\Rightarrow$ $6 + x$ is a multiple of 9 [number divisible by 9 is a multiple of 9]

$\Rightarrow$ $6 + x = 0$, or 9, or 18, ...

$\Rightarrow$ $x = -6$, or $x = 3$ or $x = 12$, ...

Since, x is a digit $\Rightarrow$ $x = 3$ (Ans.)

Example 8 :

If $5x21$ is divisible by 9; find the value of digit x.

Solution :

$\because$ $5x21$ is divisible by 9

$\Rightarrow$ $5 + x + 2 + 1$ is a multiple of 9

$\Rightarrow$ $8 + x = 0$, or 9, or 18, or 27, ...

$\Rightarrow$ $8 + x = 0$ or $8 + x = 9$ or $8 + x = 18$, ...

$\Rightarrow$ $x = -8$ or $x = 1$ or $x = 10$, ...

Since, x is a digit $\Rightarrow$ $x = 1$ (Ans.)

Example 9 :

If $24a5$ is a multiple of 3 *i.e.* divisible by 3; find the value of digit a.

Solution :

$\because$ $24a5$ is a multiple of 3

$\Rightarrow$ $2 + 4 + a + 5$ is a multiple of 3

$\Rightarrow$ $11 + a = 0, 3, 6, 9, 12, 15, 18, 21,$ ......

$\Rightarrow$ $a = -11, -8, -5, -2, 1, 4, 7, 10,$ ....

Since, a is a digit $\therefore$ $a = 1, 4$ or 7 (Ans.)

Example 10 :

If $3x72$ is divisible by 3, find the value of x.

Solution :

$\because$ $3x72$ is divisible by 3

$\Rightarrow$ $3 + x + 7 + 2$ is a multiple of 3

$\Rightarrow$ $12 + x$ is a multiple of 3

$\Rightarrow$ $12 + x = 0, 3, 6, 9, 12, 15, 18, 21, 24, ...$

$\Rightarrow$ $x = -12, -9, -6, -3, 0, 3, 6, 9, 12, ...$

Since, x is a digit $\therefore$ **$x = 0, 3, 6$ or 9** **(Ans.)**

Example 11 :

$21y8$ is a multiple of 6, find the value of digit y.

Solution :

$\because$ $21y8$ is a multiple of 6

$\Rightarrow$ $21y8$ is a multiple of 2 and 3 both

Now $21y8$ is a multiple of 3

$\Rightarrow$ $2 + 1 + y + 8$ is a multiple of 3

$\Rightarrow$ $11 + y$ is a multiple of 3

$\Rightarrow$ $11 + y = 0, 3, 6, 9, 12, 15, 18, 21, ...$

$\Rightarrow$ $y = -11, -8, -5, -2, 1, 4, 7, 10, ...$ ....(i)

$21y8$ is a multiple of 2 as it has even number 8 at its units place.

Since, y is a digit, it can have values 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 ....(ii)

Values of digit y common to equations (i) and (ii) are 1, 4 and 7

$\therefore$ **$y = 1, 4$ or 7** **(Ans.)**

Example 12 :

$13z4$ is divisible by 6, find the value of digit z.

Solution :

$13z4$ is divisible by 6, if it is divisible by 2 and 3 both.

$13z4$ is divisible by 2 as it has even number 4 at its units place.

$13z4$ is divisible by 3 $[1 + 3 + 4 = 8]$

$\Rightarrow$ $8 + z = 0, 3, 6, 9, 12, 15, 18, 21, ...$ .....(i)

$\Rightarrow$ $z = -8, -5, -2, 1, 4, 7, 10, 13, ...$ ....(ii)

Since, z is a digit, z can have values 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Values of digit z common to equations (i) and (ii) are 1, 4 and 7 **(Ans.)**

$\therefore$ **$z = 1, 4$ or 7**

Example 13 :

$2y5$ is divisible by 11, find the value of digit y.

Solution :

Sum of the digit in even place = y

and, sum of the digits in odd places = $2 + 5 = 7$

$\therefore$ Difference of the sum of the digits in even places and in odd places = $y - 7$

$2y5$ is a multiple of 11

$\Rightarrow$ $y - 7$ must be multiple of 11

$\Rightarrow$ $y - 7 = 0, 11, 22, ...$

$\Rightarrow$ $y = 7, 18, 29, ...$

$\therefore$ $y = 7$ (Ans.)

Example 14 :

$67x19$ is a multiple of 11. Find all possible values of digit x.

Solution :

Sum of the digits in even places = $1 + 7 = 8$

and, sum of the digits in odd places = $9 + x + 6 = 15 + x$

Difference between the sum of the digit in odd places and the sum of the digits in even places = $15 + x - 8 = 7 + x$

$67x19$ is a multiple of 11

$\Rightarrow$ $7 + x$ is a multiple of 11

$\Rightarrow$ $7 + x = 0, 11, 22, 33, 44, ...$

$\Rightarrow$ $x = -7, 4, 15, 26, ...$

$\therefore$ $x = 4$ (Ans.)

Example 15 :

Find the value of digit z, if $12z4$ is divisible by 4.

Solution :

$12z4$ is divisible by 4

$\Rightarrow$ $z4$ is divisible by 4

$\Rightarrow$ $10z + 4$ is divisible by 4 with unit digit 4

$\Rightarrow$ $10z + 4 = 4, 24, 44, 64, 84, 104, ...$

$\Rightarrow$ $10z = 0, 20, 40, 60, 80, 100, ...$

$\Rightarrow$ $z = 0, 2, 4, 6, 8, 10, ...$

$\therefore$ $\mathbf{z = 0, 2, 4, 6 \text{ and } 8}$ (Ans.)

EXERCISE 5(D)

For what value of digit x, is :

1. $1x5$ divisible by 3 ?
2. $31x5$ divisible by 3 ?
3. $28x6$ a multiple of 3 ?
4. $24x$ divisible by 6 ?
5. $3x26$ a multiple of 6 ?
6. $42x8$ divisible by 4 ?
7. $9142x$ a multiple of 4 ?
8. $7x34$ divisible by 9 ?
9. $5x555$ a multiple of 9 ?
10. $3x2$ divisible by 11 ?
11. $5x2$ a multiple of 11 ?